Essays in Open Macroeconomic Dynamics

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Abstract

Will capital controls enhance macro economy stability? How will the results be influenced by the exchange rate regime and monetary policy reaction? Are the consequences of policy decisions involving capital controls easily predictable, or more complicated than may have been anticipated? The first chapter of this dissertation will answer the above questions by investigating the macroeconomic dynamics of a small open economy. In recent years, these matters have become particularly important to emerging market economies, which have often adopted capital controls. We especially investigate two dynamical characteristics: indeterminacy and bifurcation. Four cases are explored, based on different exchange rate regimes and monetary policy rules.

Compared with flexible exchange rates, the fixed exchange rate regime should be a more careful policy choice due to the economic complexities. Firstly, more factors need to be considered for economic stability, such as the stickiness of prices and the elasticity of substitution between labor and consumption. Secondly, fixed exchange rate regimes with capital controls produce larger posterior probability of the indeterminate region than a flexible exchange rate regime. Thirdly, we prove the existence of Hopf bifurcation under capital controls with fixed exchange rates and current-looking monetary policy. Numerically, more types of bifurcation are detected under this policy combination.

With capital controls in place, we find that indeterminacy depends upon how the interest rate feedback to inflation and to output gap coordinate with each other in the Taylor rule. When forward-looking, both passive and positive monetary policy feedback can lead to indeterminacy.

We provide monetary policy suggestions on achieving macroeconomic stability through financial regulation and references for countries facing different choices on exchange rate regimes.

As a complement for the first chapter, the second chapter of this dissertation examines the dynamics of exchange rate as a diffusion process under flexible exchange rate regimes. When exchange rates are free floating, they are known as highly volatile, nonstationary. They are also hard to be explained and predicted by fundamentals, such as output, price level and monetary supply. This chapter uses both nonparametric method and parametric method (MLE) to estimate exchange rate as a diffusion process. In line with previous literature's finding, the nonparametric drift estimators show some nonlinearity. The result of parametric estimation shows that Geometric Brownian Motion process could be a quite good capture of the exchange rate dynamics.

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Chapter 1

Capital Control, Exchange Rate Regime and Monetary Policy: Indeterminacy and Bifurcation

1.1. Introduction

Since the Great Recession following the 2008 financial crisis, the potential problems caused by free capital movements among countries have drawn attention to the relationship between financial regulation, capital controls, and macroeconomic stability.

Some researchers support capital controls with prudential macroeconomic policy. According to that view, capital controls can mitigate systemic risk, reduce business cycle volatility, and increase macroeconomic stability. Farhi and Werning (2012, 2014) and Korinek (2011, 2014) study welfare implications of capital controls from a theoretic perspective, while Ostry et al. (2012) and Magud et al. (2012) investigate the relationship of capital controls to macroeconomic stability using empirical methods.

According to IMF's capital account openness index (Table 1), several countries in the world are imposing more strict capital controls than other countries, especially those countries which are categorized as emerging market economies. This capital account openness index ranges between 0 to 1, with 1 indicating a fully liberalized capital account. India, Bangladesh and China has the index numbers as 0.02, 0.02 and 0.09 respectively, close to the level of fully closed capital account. Brazil, Mexico, Thailand, Turkey and Algeria also have the index smaller than 0.5, showing the strictness of capital controls.

When the role of capital controls in stabilizing the economy is emphasized, however, the roles of exchange rate regimes and monetary policy should not be overlooked, especially in an open economy. According to Mundell's (1963) "impossible trinity" in international economics, an open economy can not simultaneously have independent monetary policy, fixed exchange rates, and free capital movement.[1] Besides the openness of capital account, the choice of exchange regimes and monetary policies should also be taken into important consideration to keep the economic stability.

Table 2 shows the exchange rate regimes and monetary policy frameworks of the major countries that impose strict capital controls. As shown in this table, not all of these countries choose the same exchange rate regime and monetary policy framework. Instead, some of them have floating exchange rate regime and inflation-targeting framework, such as Brazil, Mexico, India, Thailand and Turkey. While some of them choose managed exchange rates or fixed exchange rates, and monetary aggregare target, such as China, Bangladesh and Algeria.

These differences in exchange rate regimes and monetary policies show us the other intriguing facets of the international macroprudential architecture. How the choices of exchange rate regime and monetary policy influence the stabilization effect of capital control? Is it possible that the choices of exchange rate regime and monetary policy could generate instability and increased volatility, even though capital flows are controlled? What is the implication for the choices of exchange rate regime and monetary policy in an open economy that imposes capital control?

To answer the above questions, we investigate the effects of combinations of exchange rate regimes and monetary policies in stabilizing the economy under prudential macroeconomic policy with control of capital flows. Four cases are studied. They are flexible exchange rate with current looking monetary policy, fixed exchange rate with current looking monetary

[1] Mundell's (1963) "impossible trinity" is alternatively often called the "Mundell-Fleming trilemma" to recognize the relevancy of Fleming (1962).

policy, flexible exchange rate with forward looking monetary policy and fixed exchange rate with forward looking monetary policy.

In addition to different choices on exchange rate regimes and monetary policies, the property of economic stability should be further investigated, such as indeterminacy and bifurcation. What are the economic complexities that policy makers should be aware of under each case? How to make policy decisions and to what extent the policy should be adjusted are challenging questions relevant to all monetary authorities. This paper investigates the possible instability or non-uniqueness of equilibria and their relevancy to policy under capital controls.

Indeterminacy occurs if the equilibrium of an economic system is not unique, resulting in the existence of multiple equilibria. Under those circumstances, consumers' and firms' forecasts of macroeconomic variables, such as output and inflation rates, can lead to the phenomenon of "self-fulfilling prophecy." The economy can move from one equilibrium to another. A new equilibrium, driven by economic agents' beliefs, could be a better one or a worse one. If capital controls signal to people that they are protected from the risk of international financial market volatility, then the beliefs-driven equilibrium may be better. Alternatively, if imposition of capital controls produces panic and induces evasion of the controls, the equilibrium can be worse. As a result, we investigate existence of multiple equilibria in an open economy with different exchange rate regimes and monetary policies. We also empirically examine indeterminacy using Bayesian methods to estimate the probability of the indeterminacy region. We acquire the posterior estimates of parameters, the impulse responses and the variance decomposition under both fundamental shocks and sunspot shocks.

We find that the existence of indeterminacy depends upon how the interest rate feedback to inflation and output gap coordinate with each other in the Taylor rule. Our results expand the conclusions of previous literature on indeterminacy and monetary policy to the case of capital controls. When monetary policy is forward looking with capital controls, we find that both passive feedback and positive feedback can generate indeterminacy.[2]

The exchange rate regime can alter the conditions for indeterminacy. Compared with flexible exchange rates, a fixed exchange regime produces more complex conditions, depending on the stickiness of price setting and the elasticity of substitution between labor and consumption. Interestingly, the degree of openness does not play a large role in our results. This difference from previous literature is associated with the control of international capital mobility.

We introduce into our model staggered price setting. We find that when price is close to flexible with capital controls, indeterminacy is possible.

The other primary objective of our paper is to investigate existence of bifurcation phenomena in an open economy with capital controls. Bifurcation is definined to occur, if a qualitative change in dynamics occurs, when the bifurcation boundary is crossed by the deep parameters of the economy's structure. Such deep parameters are not only those of private tastes and technology, but also of monetary policy rules. Such qualitative change can be between instability and stability. But the change can also be between different kinds of instability or between different kinds of stability, such as monotonic stability and periodic damped stability, or multiperiodic damped stabilitiy. Existence of bifurcation boundaries can

[2] With passive feedback, the parameter multiplied by inflation or output gap in Taylor rule is defined to be between 0 and 1. With positive feedback, the parameter is larger than 1. Eventhough in existing literature about New Keynesian model and Taylor rule, the role of interest rate feedback to inflation is more emphasized than the role of interesit ratae feedback to output gap, we assume these two feedback have parallel weight in our model. As is shown from the Bayesian estimation results in this paper, the policy reponses to inflation and output gap do not have large difference under certain cases. This is different from a standard calibration result in previous literature.

motivate policy intervention. A slight change to the parameters of private tastes or technology or to the parameters of central bank's feedback to output and inflation can induce a fundamental change in the nature of the economy's dynamics.

We find that there can exist Hopf bifurcation under capital controls, fixed exchange rates, and current-looking monetary policy. We determine the conditions under which the monetary policy rule or private deep parameters will generate instability. We find several kinds of bifurcation numerically, when the model's parameters are estimated by Bayesian methods.

1.2. Literature Review

On the indeterminacy and monetary policy implications in macroeconomics, Benhabib and Farmer (1999) summarize the indeterminacy and sunspots in macroeconomic models, including real models, monetary models, models of endogenous growth and policy feedback. Benhabib et al. (2001) find that the conditions under which interest rate feedback to inflation generates multiple equilibria depend not only on the monetary-fiscal regime but also on the role of money in preferences and technology. They show positive monetary policy leads to multiple equilibria and passive monetary policy keeps the equilibrium uinique. Airaudo and Zanna (2012) analyze global equilibrium determinacy in a flexible-price open economy with active interest rate rules on inflation. They find that the ex ante elasticity of the real exchange rate to the policy interest rate determines the existence of endogenous cycles and chaos. This elasticity is influenced importantly by the degree of trade openness. Keating and Smith (2013) include nominal money growth in Central bank's interest rate rules and study the determinacy properties of such rules. They show that whether the equilibrium is unique or not depends on the method of aggregation. Targeting the growth rate of the true monetary aggregate will generate a unique equilibrium and this result also extends to the Divisia monetary aggregate growth rate.

On the estimation of DSGE model with indeterminacy, Lubik and Schorfheide (2004) provide the Bayesian estimation of a prototypical New Keynesian model. They construct the posterior weights for the determinacy and indeterminacy region and estimates for the propagation of fundamental and sunspot shocks. During Volcker period, the positive monetary policy put the U.S. economy into the determinacy region. Lubik and Schorfheide (2003) study the effect of sunspot shocks through endogenous forecast errors under indeterminacy. The effect of fundamental shocks on forecast erros is not uniquely determined. Under passive interest rate feedback, the response of inflation to an unanticipated interest rate cut is ambiguous. Lubik and Schorfheide (2007) use Bayesian methods to study the conduct of monetary policy in a small open economy. The results show that the central banks of Australia and New Zealand do not target exchange rates. The Bank of Canada and the Bank of England respond to exchange rate movements. Farmer et al. (2015) provide a method for solving and estimating linear rational expectations models with indeterminacy. Their method redefines a subset of expectational errors as new fundamentals, which treats the indeterminate models as determinate and possible to be solved by standard solution algorithms. Zheng and Guo (2013) estimate a small open economy DSGE model with indeterminacy. The empirical results of China show that Chinese monetary policy reacts not only to inflation and output gap, but also to the exchange rate movements.

On capital controls and exchange rate regimes, Farhi and Werning (2014) study the optimal capital controls in a small open economy with nominal rigidities. They find capital controls are desirable even when the exchange rate is flexible. In Farhi and Werning (2012), they find capital contols are effective to address risk-premium shocks and the solution depnds on the degree of nominal rigidity and the openness of the economy.

The previous literature investigating bifurcation without capital controls includes Barnett and Duzhak (2008, 2010, 2014), Barnett and Eryilmaz (2013, 2014), and the survey paper Barnett and Chen (2015). Barnett and Duzhak (2008, 2010) explore bifurcations within the class of New Keynesian models. They find the existence of Hopf bifurcation and Period Doubling bifurcation. They study cases of current-looking Taylor Rule, forward-looking Taylor Rule, Hybrid Taylor Rule, backward-looking Taylor Rule and Taylor Rule with interest rate smoothing term. Barnett and Duzhak (2014) explore bifurcations with regime switching monetary policies. Barnett and Eryilmaz (2013, 2014) find bifurcations in the open economy New Keynesian models. Compared with closed economy models, the open economy models generate more complex dynamics. In contrast, we introduce capital controls and the exchange rate peg into the open economy New Keynesian model. Without capital controls, Woodford (1986, 1992) and Franke (1992) find that capital market imperfections can lead to more complex dynamics than perfect capital markets.

1.3. Model

In light of Gali and Monacelli (2005) and Farhi and Werning (2012, 2014), our model is an open economy New Keynesian model consisting of a small open economy that imposes capital controls and chooses between flexible exchange rates and fixed exchange rates. Compared with the Mundell Fleming IS-LM-BP model, the New Keynesian model has solid micro-foundation on both the demand side and the supply side. As a result, we are able to analyze the influence of the deep structural parameters on the economy’s dynamics.

In contrast with Farhi and Werning (2012, 2014), we choose the discrete time version of the linear rational expectations model, instead of the continuous time model, to facilitate analyzing the indeterminacy and bifurcation conditions. For analyzing indeterminacy, the linear rational expectations model automatically fixes the list of predetermined variables, thereby eliminating the need to differentiate between predetermined variables and jump variables.[3] Discrete time also permits location of bifurcation boundaries in linear system, as in Barnett and Duzhak (2008, 2010, and 2014) and Barnett and Eryilmaz (2013, 2014). In addition, rational expectations allows us to differentiate between fundamental shocks and non-fundamental forecasting errors. Farmer et al. (2015) and Beyer and Farmer (2004) find methods to change the system from indeterminate to determinate by moving the non-fundamental forecasting errors. The number of those errors equals the degree of indeterminacy to the fundamental shocks set. In the rational expectations model, it is possible for beliefs to drive the economy to another path that converges to a steady state, producing a self-fulfilling prophecy. In principle, it is possible to regulate or influence those beliefs. This phenomenon is different from "animal spirit."

There is a continuum of small open economies, indexed along the unit interval. Different economies share identical preferences, technology, and market structure. Following the conventions in this literature, we use variables without *i*-index to refer to the small open economy being modelled. Variables with *i*-index refer to variables in economy *i*, among the continuum of economies making up the world economy. Variables with a star correspond to the world economy as a whole, while *j* denotes a particular good within an economy.

[3] See Sims (2002).

1.3.1. Households

A representative household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] , \tag{1}$$

where N_t denotes hours of labor, C_t is a composite consumption index defined by

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t} \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} ,$$

with $C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}, C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$.

The parameter $\varepsilon > 1$ denotes the elasticity of substitution among goods within any given country. The parameter $\alpha \in [0, 1]$ denotes the degree of home bias in preferences and is an index of openness, while $\eta > 0$ measures the elasticity of substitution between domestic and foreign goods, and γ measures the elasticity of substitution among goods produced in different countries.

The household's budget constraint takes the form

$$\begin{aligned} &\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t \left\{ Q_{t,t+1} D_{t+1} \right\} + \int_0^1 E_t \left\{ \mathcal{E}_{i,t} Q_{t,t+1}^i D_{t+1}^i \right\} di \\ &\leq W_t N_t + T_t + D_t + \int_0^1 \left(\frac{1+\tau_t}{1+\tau_t^i} \right) \mathcal{E}_{i,t} D_t^i di, \end{aligned} \tag{2}$$

where D_{t+1} is holding of the home portfolio, consisting of shares in firms. Holding of country *i*'s portfolio is D_{t+1}^{i}, while $Q_{t,t+1}$ is the price of the home portfolio, and $Q_{t,t+1}^{i}$ is the price of country *i*'s portfolio. The nominal wage is W_t. The lump-sum transfer/tax at *t* is T_t. We model the capital control, following Farhi and Werning (2014), with τ_t denoting the subsidy on capital outflows (tax on capital inflows) in home country and τ_t^i denoting the subsidy on capital outflows (tax on capital inflows) in country *i*. We assume that country *i* does not impose capital control, so that $\tau_t^i = 0$. Taxes on capital inflows are rebated as a lump sum to households. We introduce Δ and Θ to be the variables that capture the dynamics of capital control, τ_t, where $1+\tau_{t+1} \equiv \frac{\Delta_{t+1}}{\Delta_t} \equiv \frac{\Theta_{t+1}^{\sigma}}{\Theta_t^{\sigma}}$, following Farhi and Werning (2012, 2014).

After the derivation that is shown in Appendix 1, the budget constraint can be rewritten as

$$P_t C_t + E_t\left\{Q_{t,t+1} D_{t+1}\right\} + E_t\left\{\mathcal{E}_t Q_{t,t+1}^{*} D_{t+1}^{*}\right\} \le W_t N_t + T_t + D_t + (1+\tau_t)\mathcal{E}_t D_t^{*} \; . \tag{3}$$

Maximizing utility of a household subject to its budget constraint yields two Euler equations:

$$\begin{aligned} &\beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{1}{Q_{t,t+1}}\right)\right\} = 1, \\ &\beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)(1+\tau_{t+1})\left(\frac{1}{Q_{t,t+1}^{*}}\right)\right\} = 1. \end{aligned} \tag{4}$$

The log-linearized form is

$$\begin{aligned} c_t &= E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left(r_t - E_t\left\{\pi_{t+1}\right\} - \rho\right), \\ c_t &= E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left(r_t^{*} + \left[E_t\left\{e_{t+1}\right\} - e_t\right] + E_t\left\{\tau_{t+1}\right\} - E_t\left\{\pi_{t+1}\right\} - \rho\right), \end{aligned} \tag{5}$$

For the representative household in country *i*, the problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{i1-\sigma}}{1-\sigma} - \frac{N_t^{i1+\varphi}}{1+\varphi} \right], \tag{6}$$

subject to the budget constraint

$$P_t^i C_t^i + E_t\left\{Q_{t,t+1}^i D_{t+1}^i\right\} + E_t\left\{\frac{Q_{t,t+1}^{i*} D_{t+1}^{i*}}{\mathcal{E}_{i,t}}\right\} \le W_t^i N_t^i + T_t^i + D_t^i + \frac{D_t^{i*}}{\mathcal{E}_{i,t}}. \tag{7}$$

Notice that there is no capital control in country *i*.

The first order conditions also provides us with two Euler equations

$$\begin{aligned} &\beta E_t\left\{\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\left(\frac{P_t^i}{P_{t+1}^i}\right)\left(\frac{1}{Q_{t,t+1}^i}\right)\right\}=1, \\ &\beta E_t\left\{\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\left(\frac{P_t^i}{P_{t+1}^i}\right)\left(\frac{\mathcal{E}_{i,t}}{\mathcal{E}_{i,t+1}}\right)\left(\frac{1}{Q_{t,t+1}^{i*}}\right)\right\}=1, \end{aligned} \tag{8}$$

where $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^i}{P_t}$ is the real exchange rate.

Combined with the two Euler equations for the home country, we get the Backus-Smith condition,

$$C_t = \Theta_t C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}. \tag{9}$$

Taking logs on both sides and integrating over *i*, we get

$$c_t = c_t^* + \frac{1}{\sigma} q_t + \theta_t \tag{10}$$

1.3.2. Uncovered Interest Parity

The pricing equation for foreign bonds and domestic bonds are respectively

$$\begin{aligned}(R_t^*)^{-1} &= E_t\left\{Q_{t,t+1}^*\right\},\\ (R_t)^{-1} &= E_t\left\{Q_{t,t+1}\right\}.\end{aligned} \tag{11}$$

We combine them to get the Uncovered Interest Parity conditions,

$$R_t = (1+\tau_{t+1})R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$$

Taking logs on both sides, we get

$$r_t - r_t^* = E_t\left\{\tau_{t+1}\right\} + E_t\left\{e_{t+1}\right\} - e_t\ , \tag{12}$$

The effective terms of trade are $S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$.

Following Gali and Monacelli (2005), under the purchasing power parity condition, $P_{H,t} = P_{F,t}$.

The bilateral nominal exchange rate is defined by the law of one price, $P_{i,t}(j) = \mathcal{E}_{i,t} P_{i,t}^i(j)$,

where $P_{i,t}^i(j)$ is the price of country *i*'s good *j*, expressed in country *i*'s currency. It follows

that $P_{i,t} = \mathcal{E}_{i,t} P_{i,t}^i$. The nominal effective exchange rate is defined as $\mathcal{E}_t \equiv \left(\int_0^1 \mathcal{E}_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$.

The real exchange rate is defined as $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^i}{P_t}$.

We can rewrite the uncovered interest parity condition as

$$r_t - r_t^* = \sigma\left[E_t\left\{\theta_{t+1}\right\} - \theta_t\right] + \left[E_t\left\{s_{t+1}\right\} - s_t\right] + E_t\left\{\pi_{H,t+1}\right\} - E_t\left\{\pi_{t+1}^*\right\}. \tag{13}$$

1.3.3. Firms

The supply side in this paper is the same as in Gali and Monacelli (2005). Details of the derivation can be found in their paper.

A representative firm in the home country has a linear technology,

$$Y_t(j) = A_t N_t(j), \tag{14}$$

$$Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

$$Z_t \equiv \int_0^1 \frac{Y_t(j)}{Y_t} dj,$$

$$N_t \equiv \int_0^1 N_t(j)dj = \frac{Y_t Z_t}{A_t}.$$

The firm follows staggered-price setting, as in Calvo's (1983) model. Each period, $1-\omega$ of firms set new prices. The pricing decision is forward-looking. Firms set the price as a mark-up over a weighted average of expected future marginal costs. As $\omega \to 0$, the price approaches flexibility.

The dynamics of domestic inflation are given by

$$\pi_{H,t} = \beta E_t\left\{\pi_{H,t+1}\right\} + \lambda mc_t , \tag{15}$$

where

$$\lambda \equiv \frac{(1-\beta\omega)(1-\omega)}{\omega}.$$

1.3.4. Equilibrium

In this section, we assume that $\sigma = \eta = \gamma = 1$ (Cole-Obstfeld case).

1.3.4.1. Demand Side

The market clearing condition in the representative small open economy is

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j)\,di \tag{16}$$

$$= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} C_t^i di\right].$$

After the derivation in Appendix, we get

$$y_t = E_t\{y_{t+1}\} - \left(r_t - E_t\{\pi_{t+1}\} - \rho\right) - \alpha\left[E_t\{s_{t+1}\} - s_t\right] + \left[E_t\{\theta_{t+1}\} - \theta_t\right]. \tag{17}$$

1.3.4.2. Supply Side

At the steady state of the economy, we have

$$y_t = a_t + n_t. \tag{18}$$

The deviation of real marginal cost from its steady state is

$$mc_t \equiv mc_t - mc = \mu - \nu + c_t + \varphi n_t + \alpha s_t - a_t = \mu - \nu + (\varphi+1)(y_t - a_t) + \theta_t.$$

Thus at equilibrium, the dynamic equation for inflation is

$$\pi_{H,t} = \beta E_t\left\{\pi_{H,t+1}\right\} + \lambda mc_t = \beta E_t\left\{\pi_{H,t+1}\right\} + \lambda\left(\mu - \nu\right) + \lambda\left(\varphi + 1\right) y_t - \lambda\left(\varphi + 1\right) a_t + \lambda\theta_t . \tag{19}$$

1.3.4.3. Equilibrium Dynamics in Output Gap

The output gap is defined to be the following deviation of output from its natural level:

$$x_t \equiv y_t - \bar{y}_t .$$

The dynamics of the economy with capital controls and flexible exchange rates, but without monetary policy, can be written as

$$x_t = E_t\left\{x_{t+1}\right\} - \left[r_t - E_t\left\{\pi_{H,t+1}\right\} - \rho\right] + \left[E_t\left\{a_{t+1}\right\} - a_t\right] + \frac{\varphi}{\varphi + 1}\left[E_t\left\{\theta_{t+1}\right\} - \theta_t\right], \tag{20}$$

$$\pi_{H,t} = \beta E_t\left\{\pi_{H,t+1}\right\} + \lambda\left(\varphi + 1\right) x_t , \tag{21}$$

$$r_t - r_t^* = \left[E_t\left\{\theta_{t+1}\right\} - \theta_t\right] + \left[E_t\left\{s_{t+1}\right\} - s_t\right] + E_t\left\{\pi_{H,t+1}\right\} - E_t\left\{\pi_{t+1}^*\right\}.$$

If the exchange rate is fixed, then $e_{t+1} = e_t$,

$$r_t - r_t^* = \left[E_t\left\{\theta_{t+1}\right\} - \theta_t\right].$$

In the following sections of this paper, we assume that purchasing power parity holds, so that $S_t = 1$ and $\left[E_t\left\{s_{t+1}\right\} - s_t\right] = 0$.

1.3.5. Capital Control, Exchange Rate Regime and Monetary Policy: Four Cases

In this section, we summarize four cases of the dynamical system, such that the exchange rate regime can be flexible or fixed and monetary policy can be current-looking or forward-looking.

1.3.5.1. Capital Control, Flexible Exchange Rate, Current-looking Monetary Policy

This case is characterized by the following equations:

$$
\begin{aligned}
&x_t = E_t(x_{t+1}) - [r_t - E_t(\pi_{H,t+1}) - \rho] + [E_t(a_{t+1}) - a_t] + \frac{\varphi}{\varphi+1}[E_t(\theta_{t+1}) - \theta_t],\\
&\pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \lambda(\varphi+1)x_t,\\
&r_t - r_t^* = [E_t(\theta_{t+1}) - \theta_t] + E_t(\pi_{H,t+1}) - E_t(\pi_{t+1}^*),\\
&r_t = a_\pi \pi_{H,t} + a_x x_t.
\end{aligned} \tag{22}
$$

1.3.5.2. Capital Control, Fixed Exchange Rate, Current-looking Monetary Policy

This case is characterized by the following equations:

$$
\begin{aligned}
&x_t = E_t(x_{t+1}) - [r_t - E_t(\pi_{H,t+1}) - \rho] + [E_t(a_{t+1}) - a_t] + \frac{\varphi}{\varphi+1}[E_t(\theta_{t+1}) - \theta_t],\\
&\pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \lambda(\varphi+1)x_t,\\
&r_t - r_t^* = [E_t(\theta_{t+1}) - \theta_t],\\
&r_t = a_\pi \pi_{H,t} + a_x x_t.
\end{aligned} \tag{23}
$$

1.3.5.3. Capital Control, Flexible Exchange Rate, Forward-looking Monetary Policy

This case is characterized by the following equations:

$$\begin{aligned}
&x_t = E_t(x_{t+1}) - [r_t - E_t(\pi_{H,t+1}) - \rho] + [E_t(a_{t+1}) - a_t] + \frac{\varphi}{\varphi+1}[E_t(\theta_{t+1}) - \theta_t],\\
&\pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \lambda(\varphi+1)x_t,\\
&r_t - r_t^* = [E_t(\theta_{t+1}) - \theta_t] + E_t(\pi_{H,t+1}) - E_t(\pi_{t+1}^*),\\
&r_t = a_\pi E_t(\pi_{H,t+1}) + a_x E_t(x_{t+1}).
\end{aligned} \tag{24}$$

1.3.5.4. Capital Control, Fixed Exchange Rate, Forward-looking Monetary Policy

This case is characterized by the following equations:

$$\begin{aligned}
&x_t = E_t(x_{t+1}) - [r_t - E_t(\pi_{H,t+1}) - \rho] + [E_t(a_{t+1}) - a_t] + \frac{\varphi}{\varphi+1}[E_t(\theta_{t+1}) - \theta_t],\\
&\pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \lambda(\varphi+1)x_t,\\
&r_t - r_t^* = [E_t(\theta_{t+1}) - \theta_t],\\
&r_t = a_\pi E_t(\pi_{H,t+1}) + a_x E_t(x_{t+1}).
\end{aligned} \tag{25}$$

The four cases have the same IS curve and Phillips curve. The differences lie in the uncovered interest parity conditions between flexible exchange rates and fixed exchange rates, and in the interest rate feedback rule between current-looking monetary policy and forward-looking monetary policy.

It should be observed that our uncovered interest parity (UIP) condition is somewhat unusual. The usual UIP condition mainly describes the relationship between exchange rates and nominal interest rates. In our UIP condition, the nominal interest rate depends upon capital controls and upon how large the expectation of future domestic inflation will deviate from world inflation.

If the capital flow is free, so that $\tau_{t+1} = \sigma\left(\theta_{t+1} - \theta_t\right) = 0$, then under fixed exchange rates, the domestic nominal interest rate should equal the world nominal interest rate. As a result, the monetary authority loses its autonomy, in accordance with Mundell's trilemma. Second, under flexible exchange rates, the expectation of future world inflation plays a role in the dynamical system. Even though the domestic government stops targeting exchange rates and allows the exchange rate to float freely, the system is still influenced by expectations of the world inflation.

We also investigate how expectations about future domestic inflation and output gap change the results of our analysis, compared with current-looking monetary policy with the central bank setting the nominal interest rate.

1.4. Indeterminacy Conditions

In this section we investigate the indeterminacy conditions for the four cases of policy combinations. We follow the method for linear rational expectations models by Lubik and Schorfheide (2003), Lubik and Schorfheide (2004), Lubik and Marzo (2007), Sims (2002), Farmer et al. (2015), Beyer and Farmer (2004).

In Lubik and Schorfheide (2003), the indeterminacy condition is provided as follows. First, the system can be written as

$$\mathbf{\Gamma}_0 \mathbf{X}_t = \mathbf{\Gamma}_1 \mathbf{X}_{t-1} + \mathbf{\Psi} \boldsymbol{\varepsilon}_t + \mathbf{\Pi} \boldsymbol{\eta}_t \,, \tag{26}$$

where $\mathbf{X}_t$ is the $n \times 1$ vector of endogenous variables and their expectations, while $\boldsymbol{\varepsilon}_t$ is the $l \times 1$ vector of exogenous variables, and $\boldsymbol{\eta}_t$ is the $k \times 1$ vector of non-fundamental forecasting errors. Those forecast errors represent beliefs and permit self-fulfilling equilibria.

The reduced form of the above system is

$$\mathbf{X}_t = \mathbf{\Gamma}_0^{-1}\mathbf{\Gamma}_1\mathbf{X}_{t-1} + \mathbf{\Gamma}_0^{-1}\mathbf{\Psi}\boldsymbol{\varepsilon}_t + \mathbf{\Gamma}_0^{-1}\mathbf{\Pi}\boldsymbol{\eta}_t \,. \tag{27}$$

Applying generalized Schur decomposition (also called QZ decomposition) and letting $\mathbf{w}_t = \mathbf{Z}'\mathbf{X}_t$, the equation above can be written as

$$\begin{bmatrix} \mathbf{\Lambda}_{11} & \mathbf{\Lambda}_{12} \\ 0 & \mathbf{\Lambda}_{22} \end{bmatrix}\begin{bmatrix} \mathbf{w}_{1,t} \\ \mathbf{w}_{2,t} \end{bmatrix} = \begin{bmatrix} \mathbf{\Omega}_{11} & \mathbf{\Omega}_{12} \\ 0 & \mathbf{\Omega}_{22} \end{bmatrix}\begin{bmatrix} \mathbf{w}_{1,t-1} \\ \mathbf{w}_{2,t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{Q}_{1.} \\ \mathbf{Q}_{2.} \end{bmatrix}(\mathbf{\Psi}\boldsymbol{\varepsilon}_t + \mathbf{\Pi}\boldsymbol{\eta}_t)\,. \tag{28}$$

It is assumed that the following $m \times 1$ vector, $\mathbf{w}_{2,t}$, is purely explosive, where $0 \le m \le n$:

$$\mathbf{w}_{2,t} = \mathbf{\Lambda}_{22}^{-1}\mathbf{\Omega}_{22}\mathbf{w}_{2,t-1} + \mathbf{\Lambda}_{22}^{-1}\mathbf{Q}_{2.}(\mathbf{\Psi}\boldsymbol{\varepsilon}_t + \mathbf{\Pi}\boldsymbol{\eta}_t)\,.$$

A non-explosive solution of the linear rational expectation model for $\mathbf{X}_t$ exists, if $\mathbf{w}_{2,0} = \mathbf{0}$ and $\mathbf{Q}_{2.}\mathbf{\Psi}\boldsymbol{\varepsilon}_t + \mathbf{Q}_{2.}\mathbf{\Pi}\boldsymbol{\eta}_t = 0$.

By singular value decomposition of $\mathbf{Q}_{2.}\mathbf{\Pi}$, we find

$$\mathbf{Q}_{2.}\mathbf{\Pi} = \mathbf{UDV}' = \begin{bmatrix} \mathbf{U}_{.1} & \mathbf{U}_{.2} \end{bmatrix}\begin{bmatrix} \mathbf{D}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{V}'_{.1} \\ \mathbf{V}'_{.2} \end{bmatrix} = \mathbf{U}_{.1}\mathbf{D}_{11}\mathbf{V}'_{.1}\ . \tag{29}$$

The m explosive components of $\mathbf{X}_t$ generate $r \le m$ restrictions for the expectation errors. The stability condition can be rewritten as

$$\mathbf{U}_{.1}\mathbf{D}_{11}(\mathbf{V}'_{.1}\boldsymbol{\lambda}\boldsymbol{\varepsilon}_t + \mathbf{V}'_{.1}\boldsymbol{\eta}_t) = 0\,. \tag{30}$$

Let $\boldsymbol{\eta}_t = \mathbf{A}_1\boldsymbol{\varepsilon}_t + \mathbf{A}_2\boldsymbol{\zeta}_t$, where $\boldsymbol{\zeta}_t$ is a $p \times 1$ vector of sunspot shocks. The solution for the forecast errors is

$$\boldsymbol{\eta}_t = (-\mathbf{V}_{.1}\mathbf{D}_{11}^{-1}\mathbf{U}'_{.1}\mathbf{Q}_{2.}\mathbf{\Psi} + \mathbf{V}_{.2}\mathbf{M}_1)\boldsymbol{\varepsilon}_t + \mathbf{V}_{.2}\mathbf{M}_2\boldsymbol{\zeta}_t\,. \tag{31}$$

When the dimension of the vector of forecast errors, *k*, equals the number of stability restrictions, *r*, the linear rational expectations model has a unique solution. When $k > r$, there is indeterminacy (multiple stable solutions), and *k* - *r* is the degree of indeterminacy.

1.4.1. Capital Control, Flexible Exchange Rate, Current-looking Monetary Policy

Proposition 1. Under capital control, flexible exchange rate and current-looking monetary policy, there exists one degree of indeterminacy, when $\lambda(\varphi+1)(1-a_\pi)+a_x(\beta-1)>0$.

Proof. see Appendix 7.

This condition can be rewritten as

$$a_\pi + \frac{(1-\beta)}{\lambda(\varphi+1)}a_x < 1. \tag{32}$$

Indeterminacy is mainly determined by the values of a_π, a_x and λ. To satisfy this inequality, a_π must be between zero and one. If λ is large, then a_x can be large or small, so long as λ is sufficiently larger than a_x. If λ is small, a_x has to be small to generate indeterminacy.

Thus, indeterminacy is most likely to happen under passive interest rate feedback on inflation and flexible price. If price is sticky, the passive interest rate feedback on output gap in addition to inflation will also produce the multiple equilibria.

As has been analyzed by Bullard and Schaling (2006), when a_π is between zero and one, the Taylor principle is violated. In this situation, nominal interest rates rise by less than the increase in the inflation, leading a decrease in the real interest rate. This drop in real interest rate makes the output gap larger through the IS curve equation. And a rise in the output gap will increase the inflation through the Phillips curve equation. Thus, a passive monetary policy response on inflation will enlarge the inflation level, making the economy move further away from the unique equilibrium.

1.4.2. Capital Control, Fixed Exchange Rate, Current-looking Monetary Policy

Proposition 2. Under capital control, fixed exchange rates, and current-looking monetary policy, there exists one degree of indeterminacy, when

$(\varphi+1)^2(\beta+\lambda-1)+\beta[a_x(\beta-1)-\lambda a_\pi]>0$.

Proof. see Appendix 8.

This condition can be rewritten as

$$a_x(1-\beta)+\lambda a_\pi<\frac{(\varphi+1)^2(\beta+\lambda-1)}{\beta}. \tag{33}$$

Since $1-\beta>0$, to satisfy this inequality, we must have $\beta+\lambda-1>0$. In addition, a_π and a_x can not be too large. The economic intuition behind these conditions is that the price has to be flexible and interest rate feedback on inflation and output gap has to be passive to generate indeterminacy.

1.4.3. Capital Control, Flexible Exchange Rate, Forward-looking Monetary Policy

Proposition 3. Under capital control, flexible exchange rates, and forward-looking monetary policy, there exists one degree of indeterminacy, when

$$\begin{cases} a_x < \varphi + 1 \\ \lambda(\varphi+1)(1-a_\pi) + a_x(\beta - 1) > 0 \end{cases} \text{ or } \begin{cases} a_x > \varphi + 1 \\ \lambda(\varphi+1)(1-a_\pi) + a_x(\beta - 1) < 0 \end{cases}.$$

Proof. see Appendix 9.

This condition can be rewritten as

$$\begin{cases} a_x < \varphi + 1 \\ a_\pi + \dfrac{(1-\beta)}{\lambda(\varphi+1)} a_x < 1 \end{cases} \text{ or } \begin{cases} a_x > \varphi + 1 \\ a_\pi + \dfrac{(1-\beta)}{\lambda(\varphi+1)} a_x > 1 \end{cases}. \tag{34}$$

Compared with Case 1, Case 3 must consider a_x first. When a_x is small, only if a_π is between zero and one, there may exist indeterminacy. When a_x is large, there are two possibilities for indeterminacy to appear. One is that a_π is also large. The other is when λ is small.

Thus, under forward-looking monetary policy, the interest rate feedback on the output gap matters more than the interest rate feedback on the inflation. Both passive and positive feedback are possible to generate indeterminacy, depending on the corporation of these two feedback.

1.4.4. Capital Control, Fixed Exchange Rate, Forward-looking Monetary Policy

Proposition 4. Under capital control, fixed exchange rates, and forward-looking monetary policy, there exists one degree of indeterminacy, when

$$\begin{cases} a_x < \varphi + 1 \\ \lambda(\varphi+1)(\varphi+1-a_\pi) + a_x(\beta-1) > 0 \end{cases} \text{ or } \begin{cases} a_x > \varphi + 1 \\ \lambda(\varphi+1)(\varphi+1-a_\pi) + a_x(\beta-1) < 0 \end{cases}.$$

Proof. see Appendix 10.

This condition can be rewritten as

$$\begin{cases} a_x < \varphi + 1 \\ a_\pi + \dfrac{(1-\beta)}{\lambda(\varphi+1)} a_x < \varphi + 1 \end{cases} \text{ or } \begin{cases} a_x > \varphi + 1 \\ a_\pi + \dfrac{(1-\beta)}{\lambda(\varphi+1)} a_x > \varphi + 1 \end{cases}. \tag{35}$$

This case is similar to Case 3. When a_x is small, only if a_π is between zero and one, indeterminacy is possible to exist. When a_x is large, either a large a_π or a small λ will produce multiple quilibria.

Different from Case 3, the restrictions that are put on interest rate feedback on inflation and output gap are looser. This indicates that in the monetary policy parameter space, the chance to generate indeterminacy is higher under the fixed exchange rate regime.

1.5. Bifurcation Conditions

In this section, we investigate the existence of bifurcation in the system under the four policy cases. With Hopf bifurcation, the economy can converge to a stable limit cycle or diverge from an unstable limit cycle. We use the following theorem from Gandolfo (2010) to determine conditions for the existence of Hopf bifurcation.

Theorem. Consider the system, $y_{t+1} = \varphi(y_t, \alpha)$. Suppose that for each α in the relevant interval, the system has a smooth family of equilibrium points, $y_e = y_e(\alpha)$, at which the eigenvalues are complex conjugates, $\lambda_{1,2} = \theta(\alpha) \pm i\omega(\alpha)$. If there is a critical value, α_0, of the parameter α such that

(1) the eigenvalues' modulus becomes unity at α_0, but the eigenvalues are not roots of unity (from the first up to the fourth), namely

$\left|\lambda_{1,2}(\alpha_0)\right| = +\sqrt{\theta^2 + \omega^2} = 1,\ \lambda_{1,2}^{j}(\alpha_0) \neq 1$ for j = 1, 2, 3, 4,

(2) $\left.\dfrac{d\left|\lambda_{1,2}(\alpha)\right|}{d\alpha}\right|_{\alpha=\alpha_0} \neq 0$,

then there is an invariant closed curve bifurcating from α_0.

1.5.1. Capital Control, Flexible Exchange Rate, Current-looking Monetary Policy

Proposition 5. Under capital control, flexible exchange rates, and current-looking monetary policy, there would exist Hopf bifurcation, if

$[(\varphi+1)(\beta+\lambda-1)+a_x\beta]^2 + 4\lambda(\varphi+1)^2(1-a_\pi\beta) < 0$ and $(\varphi+1)(1-\beta+\lambda a_\pi)+a_x = 0$.

However, according to the meaning of the parameters, the second equation cannot be satisfied.

Proof. see Appendix 11.

1.5.2. Capital Control, Fixed Exchange Rate, Current-looking Monetary Policy

Proposition 6. Under capital control, fixed exchange rates, and current-looking monetary policy, there exists Hopf bifurcation, when

$[(\varphi+1)(\beta+\varphi)+a_x\beta]^2+4\lambda(\varphi+1)^2(\varphi+1-a_\pi\beta)<0$ and

$(1-\lambda)(\varphi+1)^2+\beta(1-\beta)(\varphi+1)+\lambda a_\pi\beta(\varphi+1)+a_x\beta=0$.

Since $\lambda=\dfrac{(1-\beta\omega)(1-\omega)}{\omega}$ with 0 < β < 1, and 0 < ω < 1, it follows that λ goes to $+\infty$, when ω approaches 0. In this case, it is possible for the second equality to hold.

Proof. see Appendix 11.

The condition will be satisfied when λ is larger than one, which happens when price is flexible.

1.5.3. Capital Control, Flexible Exchange Rate, Forward-looking Monetary Policy

Proposition 7. Under capital control, flexible exchange rates, and forward-looking monetary policy, there could exist Hopf bifurcation, if

$[(\beta-1)(\varphi+1)-\lambda(a_\pi-1)(\varphi+1)+a_x]^2+4\lambda(\varphi+1)(1-a_\pi)(\varphi+1-a_x)<0$ and

$(1-\beta)(\varphi+1)+a_x\beta=0$. However, according to the economic meaning of the parameters, the second equation cannot be satisfied with parameter values within their feasible range.

Proof. see Appendix 11.

1.5.4. Capital Control, Fixed Exchange Rate, Forward-looking Monetary Policy

Proposition 8. Under capital control, fixed exchange rates, and forward-looking monetary policy, there could exist Hopf bifurcation, if

$[(\beta+\lambda(\varphi+1-a_{\pi})-1)(\varphi+1)+a_{x}]^{2}+4\lambda(\varphi+1)(\varphi+1-a_{\pi})(\varphi+1-a_{x})<0$ and

$(1-\beta)(\varphi+1)+a_{x}\beta=0$. However, according to the meaning of the parameters, the second equation cannot be satisfied.

Proof. see Appendix 11.

1.6. Empirical Test for Indeterminacy

In this chapter, we test indeterminacy using Bayesian likelihood estimation, following Lubik and Schorfheide (2004). We compute the posterior probability of the determinate and the indeterminate regions of the parameter space. Then we estimate the parameters' posterior means and 90-percent probability intervals. We also study impulse responses of the fundamental and sunspot shocks. Last, we compute variance decompositions to study the importance of individual shocks. We use GAUSS for computations.

1.6.1. Model for Testing Indeterminacy: Four Cases

Case1: Capital Control, Flexible Exchange Rate, Current-looking Monetary Policy

In this case, the model is:

$$
\begin{aligned}
& x_t = E_{t-1}\left[x_t\right] + \eta_{1,t}, \\
& \pi_{H,t} = E_{t-1}\left[\pi_{H,t}\right] + \eta_{2,t}, \\
& x_t = E_t[x_{t+1}] - \frac{1}{\varphi+1} r_t + \frac{1}{\varphi+1} E_t[\pi_{H,t+1}] + z_t - \frac{\varphi}{\varphi+1} r_t^* + \frac{\varphi}{\varphi+1} \pi_{t+1}^* + \rho, \\
& \pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(\varphi+1) x_t, \\
& r_t = a_\pi \pi_{H,t} + a_x x_t + \varepsilon_{r,t}, \\
& z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \\
& r_t^* = \rho_R r_{t-1}^* + \varepsilon_{R,t}, \\
& \pi_t^* = \rho_\pi \pi_{t-1}^* + \varepsilon_{\pi,t},
\end{aligned}
\tag{36}
$$

which can be written as

$$
\begin{aligned}
& \mathbf{\Gamma}_0(\boldsymbol{\theta})\mathbf{s}_t = \mathbf{\Gamma}_1(\boldsymbol{\theta})\mathbf{s}_{t-1} + \mathbf{\Psi}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t + \mathbf{\Pi}(\boldsymbol{\theta})\boldsymbol{\eta}_t, \\
& \boldsymbol{\eta}_t = \mathbf{A}_1\boldsymbol{\varepsilon}_t + \mathbf{A}_2\boldsymbol{\zeta}_t,
\end{aligned}
\tag{37}
$$

with

$$
\begin{aligned}
& \mathbf{s}_t = \left[x_t, \pi_{H,t}, r_t, E_t\left[x_{t+1}\right], E_t\left[\pi_{H,t+1}\right], z_t, r_t^*, \pi_t^*\right]', \\
& \boldsymbol{\varepsilon}_t = \left[\varepsilon_{r,t}, \varepsilon_{z,t}, \varepsilon_{R,t}, \varepsilon_{\pi,t}\right]', \\
& \boldsymbol{\eta}_t = \left[\left(x_t - E_{t-1}\left[x_t\right]\right), \left(\pi_{H,t} - E_{t-1}\left[\pi_{H,t}\right]\right)\right]'.
\end{aligned}
$$

The law of motion for $\mathbf{s}_t$ can be written as:

$$
\mathbf{s}_t = \mathbf{\Gamma}_1^*(\boldsymbol{\theta})\mathbf{s}_{t-1} + \mathbf{B}_1(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t + \mathbf{B}_2(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t + \mathbf{\Pi}^*(\boldsymbol{\theta})\mathbf{V}_{.2}(\boldsymbol{\theta})\mathbf{M}_\zeta\boldsymbol{\zeta}_t\,. \tag{38}
$$

Details about the above matrices can be found in Lubik and Schorfheide (2004).

Case 2: Capital Control, Fixed Exchange Rate, Current-looking Monetary Policy

In this case, the model is:

$$
\begin{aligned}
& x_t = E_{t-1}\left[x_t\right] + \eta_{1,t}, \\
& \pi_{H,t} = E_{t-1}\left[\pi_{H,t}\right] + \eta_{2,t}, \\
& x_t = E_t[x_{t+1}] - \frac{1}{\varphi+1} r_t + E_t[\pi_{H,t+1}] + z_t - \frac{\varphi}{\varphi+1} r_t^* + \rho, \\
& \pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(\varphi+1)x_t, \\
& r_t = a_\pi \pi_{H,t} + a_x x_t + \varepsilon_{r,t}, \\
& z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \\
& r_t^* = \rho_R r_{t-1}^* + \varepsilon_{R,t},
\end{aligned}
\tag{39}
$$

which can be written as (37) with

$$
\begin{aligned}
& \mathbf{s}_t = \left[x_t, \pi_{H,t}, r_t, E_t\left[x_{t+1}\right], E_t\left[\pi_{H,t+1}\right], z_t, r_t^*\right]', \\
& \boldsymbol{\varepsilon}_t = \left[\varepsilon_{r,t}, \varepsilon_{z,t}, \varepsilon_{R,t}\right]', \\
& \boldsymbol{\eta}_t = \left[\left(x_t - E_{t-1}\left[x_t\right]\right), \left(\pi_{H,t} - E_{t-1}\left[\pi_{H,t}\right]\right)\right]'.
\end{aligned}
$$

Case 3: Capital Control, Flexible Exchange Rate, Forward-looking Monetary Policy

In this case, the model is:

$$
\begin{aligned}
&x_t = E_{t-1}\left[x_t\right] + \eta_{1,t}, \\
&\pi_{H,t} = E_{t-1}\left[\pi_{H,t}\right] + \eta_{2,t}, \\
&x_t = E_t[x_{t+1}] - \frac{1}{\varphi+1} r_t + \frac{1}{\varphi+1} E_t[\pi_{H,t+1}] + z_t - \frac{\varphi}{\varphi+1} r_t^* + \frac{\varphi}{\varphi+1} \pi_{t+1}^* + \rho, \\
&\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(\varphi+1) x_t, \\
&r_t = a_\pi E_t[\pi_{H,t+1}] + a_x E_t[x_{t+1}] + \varepsilon_{r,t}, \\
&z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \\
&r_t^* = \rho_R r_{t-1}^* + \varepsilon_{R,t}, \\
&\pi_t^* = \rho_\pi \pi_{t-1}^* + \varepsilon_{\pi,t},
\end{aligned} \tag{40}
$$

which can be written as (37) with

$$
\begin{aligned}
&\mathbf{s}_t = \left[x_t, \pi_{H,t}, r_t, E_t\left[x_{t+1}\right], E_t\left[\pi_{H,t+1}\right], z_t, r_t^*, \pi_t^*\right]', \\
&\boldsymbol{\varepsilon}_t = \left[\varepsilon_{r,t}, \varepsilon_{z,t}, \varepsilon_{R,t}, \varepsilon_{\pi,t}\right]', \\
&\boldsymbol{\eta}_t = \left[\left(x_t - E_{t-1}\left[x_t\right]\right), \left(\pi_{H,t} - E_{t-1}\left[\pi_{H,t}\right]\right)\right]'.
\end{aligned}
$$

Case 4: Capital Control, Fixed Exchange Rate, Forward-looking Monetary Policy

In this case, the model is:

$$
\begin{aligned}
&x_t = E_{t-1}\left[x_t\right] + \eta_{1,t}, \\
&\pi_{H,t} = E_{t-1}\left[\pi_{H,t}\right] + \eta_{2,t}, \\
&x_t = E_t[x_{t+1}] - \frac{1}{\varphi+1} r_t + E_t[\pi_{H,t+1}] + z_t - \frac{\varphi}{\varphi+1} r_t^* + \rho, \\
&\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(\varphi+1) x_t, \\
&r_t = a_\pi E_t[\pi_{H,t+1}] + a_x E_t[x_{t+1}] + \varepsilon_{r,t}, \\
&z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \\
&r_t^* = \rho_R r_{t-1}^* + \varepsilon_{R,t},
\end{aligned} \tag{41}
$$

which can be written as (37) with

$$\mathbf{s}_t = \left[x_t, \pi_{H,t}, r_t, E_t\left[x_{t+1}\right], E_t\left[\pi_{H,t+1}\right], z_t, r_t^* \right]',$$

$$\boldsymbol{\varepsilon}_t = \left[\varepsilon_{r,t}, \varepsilon_{z,t}, \varepsilon_{R,t} \right]',$$

$$\boldsymbol{\eta}_t = \left[\left(x_t - E_{t-1}\left[x_t\right]\right), \left(\pi_{H,t} - E_{t-1}\left[\pi_{H,t}\right]\right) \right]'.$$

1.6.2. Data Description: China 1999(1) to 2004(4) and 2011(4) to 2017(3)

We choose China as the example of a small open economy which imposes capital control and has the time periods of two exchange rate regimes. There are several reasons for this choice. First, for the common examples of small open economies, such as Australia, Canada, New Zealand and the UK, they have seldom imposed controls on capital flows in history. Second, among the major emerging market economies which impose capital control as discussed in the introduction, not many of them have experienced both flexible exchange rate regimes and fixed exchange rate regimes during the periods of capital contols. Thus, we take China as the sample economy for our estimation. As shown in the figure of China/U.S. foreign exchange rate, we take the period of 1999(1) to 2004(4) as the time of fixed exchange rate regimes. And 2011(4) to 2017(3) are selected as the sample for flexible exchange rate regimes. The models of Case 1 and Case 3 are fitted to the data of 2011(4) to 2017(3). Case 2 and Case 4 are fitted to the data of 1999(1) to 2004(4).

We use quarterly data from the database, FRED, of the Federal Reserve Bank of St. Louis. The output level is measured as the real Gross Domestic Product (GDP). We take the HP trend of real GDP as the potential output level of China. The output gap is calculated as the log of real GDP minus the log of HP trend. Inflation is measured as the log of Consumer Prive Index (CPI). The nominal interest rate is the central bank rates for China.

The prior means and densities are chosen based on previous research, such as Lubik and Frank (2004), Lubik and Frank (2007) and Zheng and Guo (2013). Notice that the period of 1999(1) to 2004(4) in China is the time under passive interest rate feedback to inflation, so we set the prior mean of a_π to be 0.8. While during 2011(4) to 2017(3) in China, the interest rate feedback to inflation is positive. We set the prior mean of a_π to be 1.1.

Following previous literature, the monetary policy parameters follow Gamma distribution. The parameter for price sticknes follows Beta distribution. The correlations between shocks follow Normal distribution. The exogenous shocks follow Inverse Gamma distribution.

1.6.3. Parameter Estimation

Under the flexible exchange rate regime and current-looking monetary policy in China, the posterior mean of interest rate feedback to inflation a_π is 2.33. And the interest rate feedback to output gap a_x is 0.46. If inflation increases by 1%, the nominal interest rate raises by 233 base points. If real output is 1% higher than its potential level, the nominal interest rate responses by increasing 46 base points. There exist correlations between fundamental shocks. And indeterminacy can influence the transmission of structural shocks related to monetary policy, technology, foreign interest rate and foreign inflation. ω equals to 0.99, which reflects that the price is very sticky.

Under the fixed exchange rate regime and current-looking monetary policy in China, the posterior mean of interest rate feedback to inflation a_π is 0.22. And the interest rate feedback to output gap a_x is 0.22. Unlike the standard calibration results of a New Keynesian model, the interest rate feedback to inflation and output gap has similar weight in the monetary policy response function.

Under the flexible exchange rate regime and forward-looking monetary policy in China, the posterior mean of interest rate feedback to inflation a_π is 0.59. And the interest rate feedback to output gap a_x is 0.42. These two feedback has similar weight in the monetary policy response function. The correlations between fundamental shocks exist. And the transmission of these fundamental shocks is also influenced by the indeterminacy. Price is very sticky.

Under the fixed exchange rate regime and forward-looking monetary policy in China, the posterior mean of interest rate feedback to inflation a_π is 0.31. And the interest rate feedback to output gap a_x is 0.29. These two feedback has similar weight in the monetary policy response function.

1.6.4. Posterior Probability of the Determinate and the Indeterminate Regions

The posterior probabilities of the determinate and indeterminate regions indicate that indeterminacy is a greater risk under fixed exchange rate regimes than under flexible exchange rate regimes.

The forward-looking monetary policy reduces the probability of indeterminate region under flexible exchange rate regimes. However, under fixed exchange rate regimes, a forward-looking monetary policy increases the probability of indeterminacy.

1.6.5. Impulse Responses

Under flexible exchange rate regime and current-looking monetary policy, an unanticipated tightening of monetary policy reduces output and inflation. Interest rate increases immediately. One unit positive technology shock increases output, inflation and interest rate permanently. In response to foreign interest rate shock, output, inflation and interest rate increase permanently. This increase of domestic interest rate in response to foreign interest shock shows the dependence of monetary policy, even under the controlled capital flows and flexible exchange rates. Foreign inflation shock only takes effect under flexible exchange rate regimes. Under foreign inflation shock, output, inflation and interest rate decrease permanently. In response to sunspot driven inflationary expectation, output firstly decreases and then increases permanently. Interest rate also increases permanently. It firstly jumps up and then drops to a lower positive level.

Under the same exchange rate regime, there are some differences in the impulse responses when monetary policy is forward looking. First, all the responses of output, inflation and interest rate go back to their steady states in the long run. Second, under a positive technology shock, output and interest rate firstly increase and then decrease. Third, inflation decreases in response to both technology shock and foreign interest rate shock. This response of inflation to foreign interest rate shock is different from that under current-looking monetary policy. Last, under foreign inflation shock, output, inflation and interest rate increase, rather than decrease. This is different from their responses under current-looking monetary policy.

Under fixed exchange rate regime and current-looking monetary policy, output, inflation and interest rate increase in response to an unanticipated tightening of monetary policy. Under technology shock and foreign interest rate shock, output increases permanently. Inflation and interest rate decrease permanently. This response of domestic interest rate in the opposite direction of foreign interest rate shows the monetary policy independence under capital controls and fixed exchange rate regimes, in line with the Mundell-Fleming trilemma. The sunspot driven inflationary expectation increases output, inflation and interest rate. This reflects the fact of self-fulfilling prophecy.

When the monetary policy is forward-looking under fixed exchange rate regime, technology shock increases output and decreases inflation and interest rate. Inflation and interest rate first increase and then decrease in response to foreign interest rate shock. These are different from their responses under current-looking monetary policy. And it also shows that the monetary policy is not completely independent when it is forward-looking, which slightly deviates from the Mundell-Fleming trilemma.

1.6.6. Variance Decomposition

The variance decomposition results provide the contributions of each shock to the fluctuations in output gap, inflation and interest rate.

Under the flexible exchange rate regime and current-looking monetary policy in China, foreign interest rate shock contributes to most of the fluctuations in output gap (45.51%), inflation (56.84%) and interest rate (32.28%). Foreign inflation shock also makes important contribution. It explains the fluctuations in output gap (30.16%), inflation (21.38%) and interest rate (20.81%).

When monetary policy is forward-looking under the flexible exchange rate regime, foreign inflation shock contributes to most of the fluctuations in output gap (51.1%) and inflation (63.15%). Monetary policy shock contributes to 79.61% of fluctuation in interest rate.

Under the fixed exchange rate regime and current-looking monetary policy, technology shock contributes to most of the fluctuations in output gap (45.53%), inflation (69.06%) and interest rate (58.23%). Foreign interest rate shock explains the fluctuations in output gap (46.61%), inflation (28.55%) and interest rate (24.79%).

When monetary policy is forward-looking under the fixed exchange rate regime, technology shock still contributes to most of the fluctuations in output gap (61.76%), inflation (91.8%) and interest rate (62.68%).

1.6.7. Robustness: United States 1954(3) to 1971(4) and 1980(1) to 1985(4)

Data Description

As the robustness test, we use quarterly postwar U.S. data on output, inflation, and nominal interest rates from the database, FRED. We consider the following two sample periods: 1954(3) to 1971(4) and 1980(1) to 1985(4). These two periods are during and right after the Bretton Woods System, during which the U.S. was imposing capital controls. During the first sample period, the U.S. dollar was tied to gold, while after the Bretton Woods System had ended, the link to gold was terminated. The U.S. dollar became a freely floating fiat currency. The models of Case 1 and Case 3 are fitted to the data of 1980(1) to 1985(4), while Case 2 and Case 4 are fitted to the data of 1954(3) to 1971(4).

The output gap is calculated as the log of real GDP minus the log of real potential GDP. Inflation is calculated as the log of the CPI. The nominal interest rate is the Effective Federal Funds Rate.

Notice that the period of 1954(3) to 1971(4) in U.S. is the time under passive interest rate feedback to inflation, so we set the prior mean of a_π to be 0.8. While during 1980(1) to 1985(4) in U.S., the interest rate feedback to inflation is positive. We set the prior mean of a_π to be 1.1.

Parameter Estimation

As the robust test, under flexible exchange rate regimes and current-looking monetary policy, the U.S. results show some difference. The posterior mean of interest rate feedback to inflation a_π is 6.18. And the interest rate feedback to output gap a_x is 0.28. Indeterminacy influences the transmission of monetary policy shock and foreign inflation shock. ω is 0.37, which shows that the price is flexible.

The estimation results for U.S. under flexible exchange rate regimes and forward-looking monetary policy shows that the posterior mean of interest rate feedback to inflation a_π is 1.66. And the interest rate feedback to output gap a_x is 0.41. Indeterminacy influences the transmission of monetary policy shock and foreign inflation shock. ω is 0.96, which shows that the price is sticky.

Compared with China, under fixed exchange rate regimes and current-looking monetary policy in U.S., the posterior mean of interest rate feedback to inflation a_π is 6.91. And the interest rate feedback to output gap a_x is 0.33. It is a little out of expectation that the interest rate feedback to inflation is positive, since this feedback is assumed to be passive during the Bretton Wood System. There exist some correlations between fundamental shocks. And indeterminacy can influence the transmission of structural shocks related to monetary policy, technology and foreign interest rate. ω is 0.23, which shows that the price is flexible.

Under fixed exchange rate regimes and forward-looking monetary policy, the U.S. results show that the posterior mean of interest rate feedback to inflation a_π is 1.25. And the interest rate feedback to output gap a_x is 0.38. There exist some correlations between fundamental shocks. And indeterminacy can influence the transmission of structural shocks related to monetary policy, technology and foreign interest rate.

Posterior Probability of the Determinate and the Indeterminate Regions

Under flexible exchange rate and current-looking monetary policy, the probability of indeterminacy region of US is smaller than that of China. Under flexible exchange rate and forward-looking monetary policy, the probability of indeterminacy region of US is larger than that of China. Under fixed exchange rate and current-looking monetary policy, the probability of indeterminacy region of US is larger than that of China. Under fixed exchange rate and forward-looking monetary policy, the probability of indeterminacy region of US is larger than that of China.

Impulse Responses

Under both flexible exchange rate and fixed exchange rate with current-looking monetary policy, the responses of output gap, inflation and interest rate under foreign interest rate shock are different from those of China. In US, the inflation and the interest rate firstly decrease and then increase. The response of interest rate reflects a certain degree of monetary policy independence. The US economy is less influenced by the sunspot shock.

Under flexible exchange rate and forward-looking monetary policy, the impulse responses of output gap, inflation and interest rate to foreign inflation shock are different from those of China. All of them decrease under this shock. The US economy is less influenced by the sunspot shock.

Under fixed exchange rate and forward-looking monetary policy, the impulse responses of output gap, inflation and interest rate to foreign interest rate shock are different from those of current-looking monetary policy. Different from the case of China, the interest rate directly decreases in response to foreign interest rate shock. Different from the other three cases of US, the sunspot shock takes effect on output gap, inflation and interest rate.

Variance Decomposition

As the robustness test, the U.S. variance decomposition results shows that under flexible exchange rate regimes and current-looking monetary policy, monetary policy shock explains 37.17% of fluctuation in output gap. Technology shock contributes to 38.22% of fluctuation in inflation and 39.11% of fluctuation in interest rate. Foreign interest rate shock contributes to 29.79% of fluctuation in output gap, 33.72% of fluctuation in inflation and 34.75% of fluctuation in interest rate. When monetary policy is forward-looking, foreign inflation shock contributes to most of the fluctuations in output gap (57.16%), inflation (52.49%) and interest rate (51.4%).

Under fixed exchange rate regimes and current-looking monetary policy, foreign interest rate shock contributes to most of the fluctuations in output gap (63.52%), inflation (65.83%) and interest rate (67.97%). When monetary policy is forward-looking, foreign interest rate shock explains the fluctuations in output gap (41.38%), inflation (54%) and interest rate (52.5%).

1.7. Numerical Bifurcation Analysis

In this section, we detect bifurcation numerically. In line with our former analysis, we find numerically that bifurcation exists under fixed exchange rate regimes and current-looking monetary policy. We used MatContM and Mathematica to perform the computations. We find that at certain values of the deep parameters, the dynamical system becomes unstable. Several kinds of bifurcation appear at those values, both when computed forward and backward at those values. Notice that a_x and a_π are the interest rate feedback in the Taylor rule to the output gap and to inflation respectively. We find that when capital controls are imposed, policy makers should be cautious, when adjusting the nominal interest rate under fixed exchange rate regimes with current-looking monetary policy.

To explore bifurcation phenomena, we define a and b such that

$$a = 1 + \frac{a_x}{\varphi + 1} + \frac{\varphi + 2}{\beta}, \tag{42}$$

$$b = \frac{1 + a_\pi \lambda}{\beta} + \frac{(\varphi + 1)(1 - \lambda)}{\beta^2} + \frac{a_x}{\beta(\varphi + 1)}. \tag{43}$$

As the results summarized in Table 37, at *a* = 4.88 and *b* = 3.88, we find a branch point and it is unstable improper node. Selecting this branch point as initial point and keep computing backward, we get a bifurcation where another branch point shows up. At *a* = 4.85, *b* = 3.85, we find the same types of bifurcation as above. At *a* = 4.85, *b* = 1, we detect a neutral saddle and it is unstable improper node. At *a* = 4.85, *b* = -5.85, we find a period doubling point and it is saddle point.

Since the values of *a* and *b* are also functions of the monetary policy parameters and deep structural parameters, it is able to be identified that certain values of monetary policy a_x and a_π will lead the economy into instability. And these values should be where the policy makers be careful with.

1.8. Conclusion

We investigated the dynamical properties and stability of the macroeconomy under capital controls. Conditional on different exchange rate regimes and monetary policies, we classified our analysis into four different cases. We show that under certain conditions of the deep parameters and monetary policy parameters, the macro economy will have multiple equilibria and can be unstable, especially under fixed exchange rate regimes and current-looking monetary policy. Monetary authorities need to be cautious, when they make policy decisions with capital controls. Only when taking these complexities into consideration, can macro-prudential policy with capital controls play its role in stabilizing the macro economy. The common view that capital controls can provide a simple solution to difficult problems can be seriously misguided, producing unanticipated risk. The economy could become trapped in a worse equilibrium or in an instability region, leading the economy onto a volatile path.

Under capital control, policy makers could move the economic system from indeterminate equilibria to determinate equilibrium by adjusting non-fundamental forecasting error to the set of fundamental shocks. One method, would be by changing people's belief. An altnermative method, more directly under government control, would be by changing the value of policy parameters to move the system from an instability region to stability region.

We assume purchasing power parity, thereby removing the dynamics of terms of trade and exchange rates from the dynamical systems. Extensions of our model could permit solving for the dynamics of exchange rates and terms of trade. In addition, some of our results produce indeterminacy, and some produce deterministic business cycles without stochastic shocks. Extensions to explore stability in a stochastic economic system is a future research goal.

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1.10. Tables and Figures

Table 1: Capital Account Openness Index

Country	Capital Account Openness Index
Algeria	0.11
Bangladesh	0.02
Brazil	0.41
China	0.09
India	0.02
Mexico	0.39
Thailand	0.3
Turkey	0.39

Notes: Overall Openness Index is the unweighted average of the openness of twelve types of asset categories: equity, bond, money market, collective investment, derivates and other instruments, commercial credit, financial credit, direct investment, direct investment liquidation, guarantees, real estate and personal capital transaction. Range between 0 to 1 (1 indicates fully liberalized).

Source: International Monetary Fund, April 2016

Table 2: Exchange Rate Regime and Monetary Policy Framework

Country	Exchange Rate Regime	Monetary Policy Framework
Algeria	Other managed arrangement	Monetary aggregate target
Bangladesh	Stabilized arrangement	Monetary aggregate target
Brazil	Floating	Inflation-targeting framework
China	Other managed arrangement	Monetary aggregate target
India	Floating	Inflation-targeting framework
Mexico	Free floating	Inflation-targeting framework
Thailand	Floating	Inflation-targeting framework
Turkey	Floating	Inflation-targeting framework

Source: IMF, April 2016

Table 3: Indeterminacy Conditions

Policies	Indeterminacy conditions
Capital control, Flexible exchange rates, Current-looking monetary policy	$\lambda(\varphi+1)(1-a_{\pi})+a_{x}(\beta-1)>0$
Capital control, Fixed exchange rates, Current-looking monetary policy	$(\varphi+1)^{2}(\beta+\lambda-1)+\beta[a_{x}(\beta-1)-\lambda a_{\pi}]>0$
Capital control, Flexible exchange rates, Forward-looking monetary policy	$\begin{cases} a_{x}<\varphi+1 \\ \lambda(\varphi+1)(1-a_{\pi})+a_{x}(\beta-1)>0 \end{cases}$ or $\begin{cases} a_{x}>\varphi+1 \\ \lambda(\varphi+1)(1-a_{\pi})+a_{x}(\beta-1)<0 \end{cases}$
Capital control, Fixed exchange rates, Forward-looking monetary policy	$\begin{cases} a_{x}<\varphi+1 \\ \lambda(\varphi+1)(\varphi+1-a_{\pi})+a_{x}(\beta-1)>0 \end{cases}$ or $\begin{cases} a_{x}>\varphi+1 \\ \lambda(\varphi+1)(\varphi+1-a_{\pi})+a_{x}(\beta-1)<0 \end{cases}$

Table 4: Bifurcation Conditions

Policies	Bifurcation conditions	Possible
Case 1	$[(\varphi+1)(\beta+\lambda-1)+a_x\beta]^2+4\lambda(\varphi+1)^2(1-a_\pi\beta)<0$ $(\varphi+1)(1-\beta+\lambda a_\pi)+a_x=0$	No
Case 2	$[(\varphi+1)(\beta+\varphi)+a_x\beta]^2+4\lambda(\varphi+1)^2(\varphi+1-a_\pi\beta)<0$ $(1-\lambda)(\varphi+1)^2+\beta(1-\beta)(\varphi+1)+\lambda a_\pi\beta(\varphi+1)+a_x\beta=0$	Yes
Case 3	$[(\beta-1)(\varphi+1)-\lambda(a_\pi-1)(\varphi+1)+a_x]^2+4\lambda(\varphi+1)(1-a_\pi)(\varphi+1-a_x)<0$ $(1-\beta)(\varphi+1)+a_x\beta=0$	No
Case 4	$[(\beta+\lambda(\varphi+1-a_\pi)-1)(\varphi+1)+a_x]^2+4\lambda(\varphi+1)(\varphi+1-a_\pi)(\varphi+1-a_x)<0$ $(1-\beta)(\varphi+1)+a_x\beta=0$	No

Figure 1: China/U.S. Foreign Exchange Rate

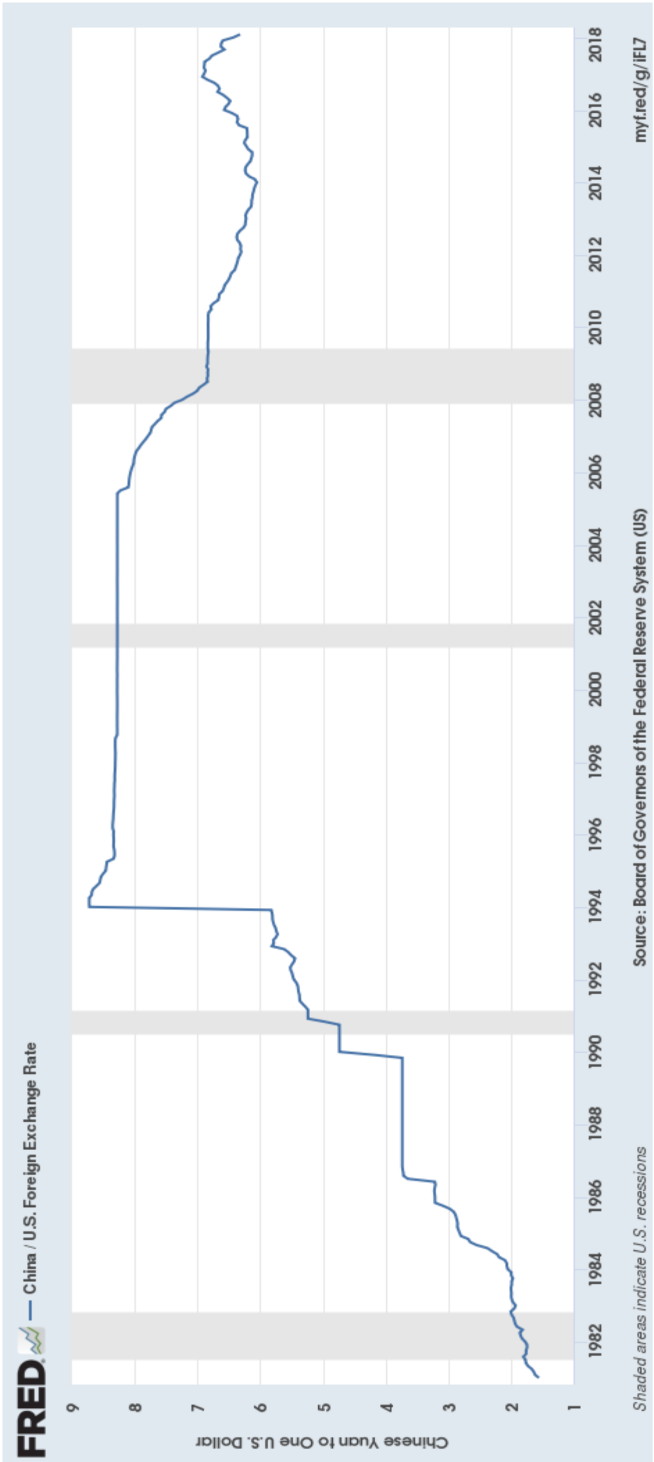


Figure 2: Foreign Assets in the U.S. (Net Capital Inflow)

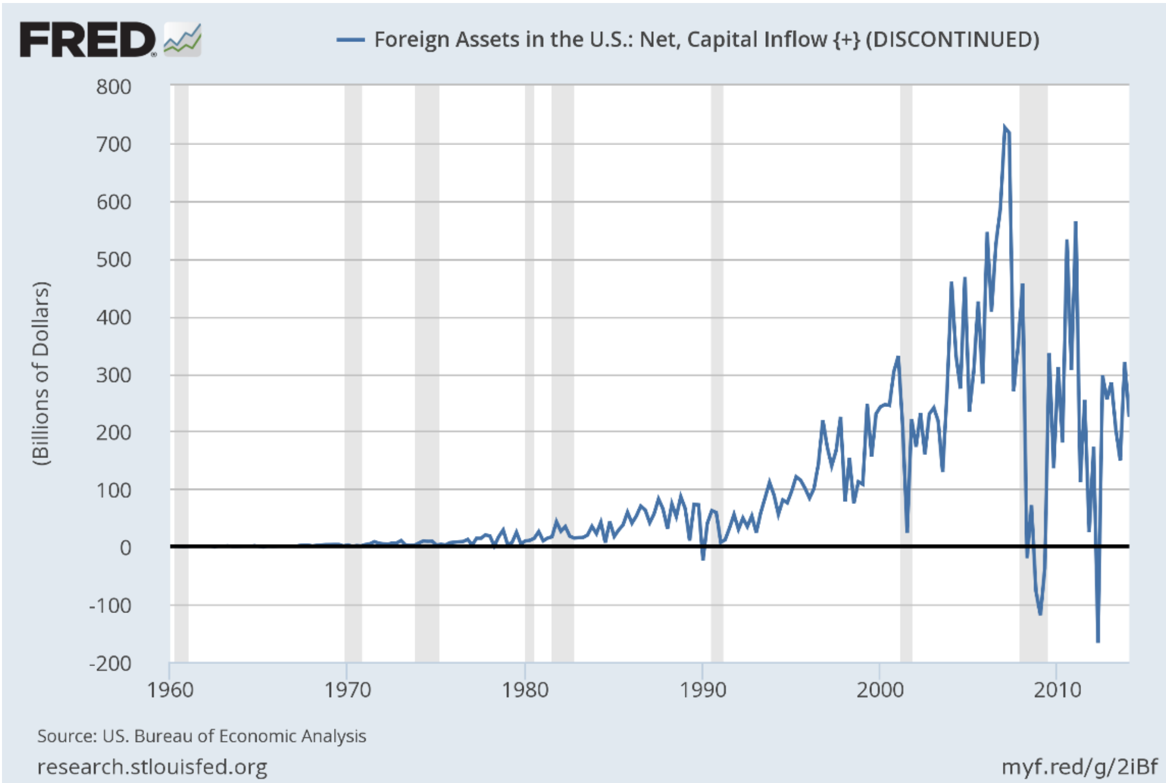


Figure 3: Gold Fixing Price in U.S. Dollars

Table 5: Prior Distributions-Flexible Exchange Rate, Current-looking Monetary Policy, China

Parameter	Density	Prior Mean	Prior Standard Deviation
a_{π}	Gamma	1.1000	0.5000
a_{x}	Gamma	0.2500	0.1500
ω	Beta	0.8000	0.1000
π^{*}	Gamma	4.0000	2.0000
r^{*}	Gamma	2.0000	1.0000
φ	Gamma	2.0000	0.7500
ρ_{z}	Beta	0.9000	0.1000
ρ_{R}	Beta	0.5000	0.2000
ρ_{π}	Beta	0.7000	0.1000
ρ_{zR}	Normal	0.0000	0.4000
$\rho_{z\pi}$	Normal	0.0000	0.4000
$\rho_{R\pi}$	Normal	0.0000	0.4000
$M_{r\zeta}$	Normal	0.0000	1.0000
$M_{z\zeta}$	Normal	0.0000	1.0000
$M_{R\zeta}$	Normal	0.0000	1.0000
$M_{\pi\zeta}$	Normal	0.0000	1.0000
σ_{r}	Inverse Gamma	0.2500	4.0000
σ_{z}	Inverse Gamma	0.8000	4.0000

σ_R	Inverse Gamma	0.3000	4.0000
σ_π	Inverse Gamma	0.3000	4.0000
σ_ζ	Inverse Gamma	0.2000	4.0000

Table 6: Parameter Estimation Results-Flexible Exchange Rate, Current-looking Monetary Policy, China

Parameter	Mean	Standard Deviation	90 % Posterior Interval Lower Bound	90 % Posterior Interval Upper Bound
a_π	2.3296	0.0775	2.2824	2.4474
a_x	0.4600	0.0274	0.4109	0.4776
ω	0.9888	0.0002	0.9886	0.9891
π^*	4.7341	0.0011	4.7328	4.7348
r^*	0.2246	0.0573	0.1762	0.2697
φ	0.7383	0.1432	0.6542	0.9976
ρ_z	0.9778	0.0057	0.9653	0.9804
ρ_R	0.9707	0.0095	0.9683	0.9876
ρ_π	0.6387	0.0174	0.6317	0.6653
ρ_{zR}	0.9596	0.0181	0.9434	0.9740
$\rho_{z\pi}$	-0.6645	0.0419	-0.7095	-0.6481

$\rho_{R\pi}$	-0.4416	0.0556	-0.5323	-0.4128
$M_{r\zeta}$	-0.2613	0.1750	-0.6390	-0.1439
$M_{z\zeta}$	-0.6844	0.1165	-0.7770	-0.4905
$M_{R\zeta}$	-0.6962	0.1780	-0.8476	-0.3938
$M_{\pi\zeta}$	0.3297	0.1516	0.2239	0.5487
σ_r	0.6276	0.0171	0.5985	0.6384
σ_z	0.2537	0.0210	0.2359	0.2996
σ_R	0.5105	0.0256	0.4827	0.5256
σ_π	0.3867	0.0146	0.3853	0.3955
σ_ζ	0.2579	0.0232	0.2604	0.2687

Notes: The posterior summary statistics are calculated by the Metropolis-Hastings algorithm.

Table 7: Determinacy versus Indeterminacy-Flexible Exchange Rate, Current-looking Monetary Policy, China

Probability	
Determinacy	Indeterminacy
0.5709	0.4291

Notes: The posterior probabilities are calculated by the Metropolis-Hastings algorithm.

Table 8: Variance Decomposition-Flexible Exchange Rate, Current-looking Monetary Policy, China

	Output Gap	Inflation	Interest Rate
Monetary Policy Shock	0.0418 [0.0273, 0.0496]	0.0001 [0.0000, 0.0002]	0.3205 [0.1657, 0.3458]
Technology Shock	0.1405 [0.1251, 0.1444]	0.1536 [0.1353, 0.1988]	0.0984 [0.0881, 0.1342]
Foreign Interest Rate Shock	0.4551 [0.4060, 0.4901]	0.5684 [0.4904, 0.7671]	0.3228 [0.2879, 0.3898]
Foreign Inflation Shock	0.3016 [0.2204, 0.3642]	0.2138 [0.0597, 0.2939]	0.2081 [0.1578, 0.2586]
Sunspot Shock	0.0610 [0.0402, 0.0681]	0.0641 [0.0183, 0.0803]	0.0502 [0.0457, 0.0658]

Notes: This table reports the posterior mean and 90% probability intervals.

Figure 4: Impulse Responses-Flexible Exchange Rate, Current-looking Monetary Policy, China

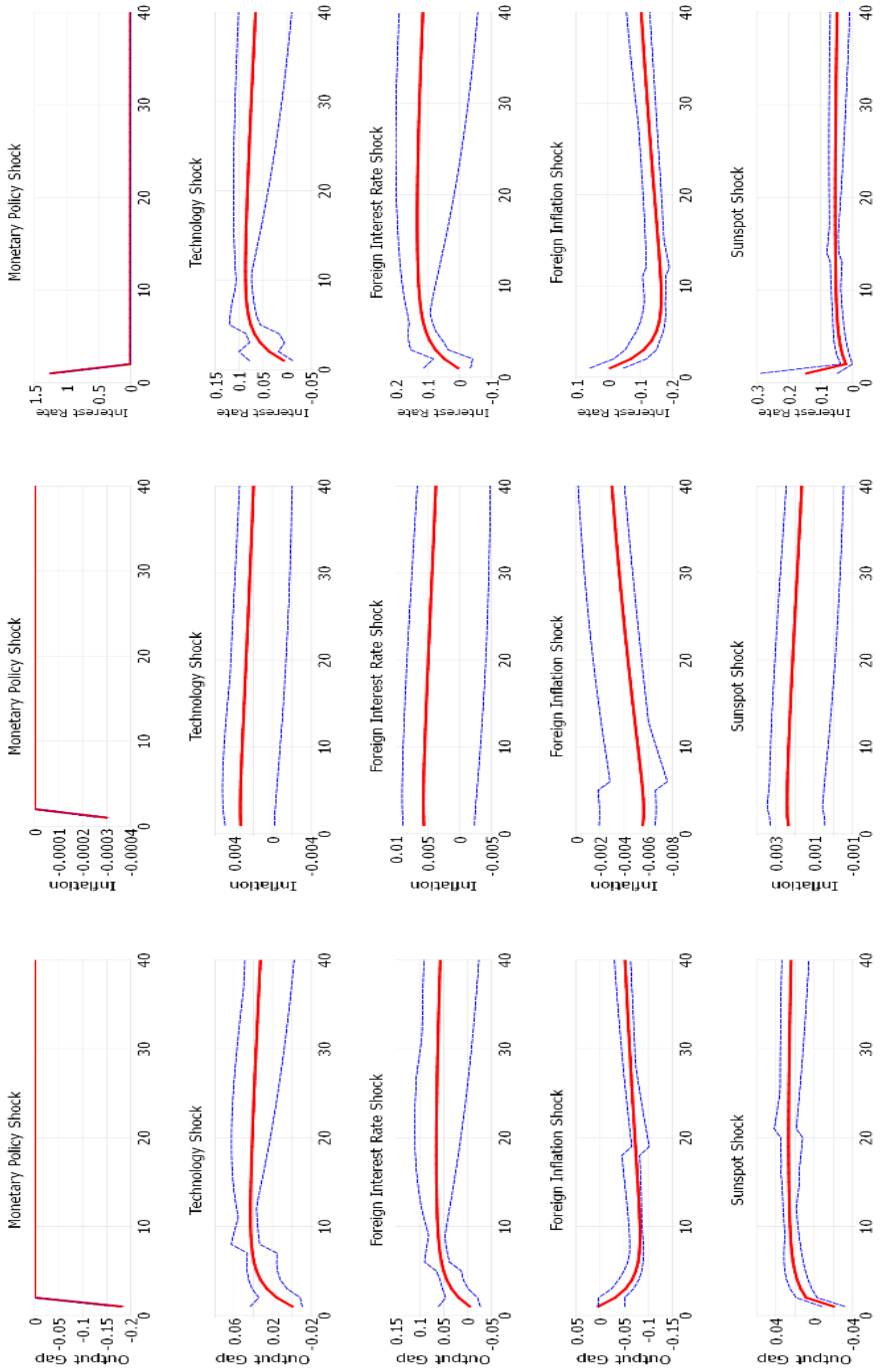


Table 9: Prior Distributions-Fixed Exchange Rate, Current-looking Monetary Policy, China

Parameter	Density	Prior Mean	Prior Standard Deviation
a_π	Gamma	0.8000	0.5000
a_x	Gamma	0.2500	0.1500
ω	Beta	0.8000	0.1000
π^*	Gamma	4.0000	2.0000
r^*	Gamma	2.0000	1.0000
φ	Gamma	2.0000	0.7500
ρ_z	Beta	0.9000	0.1000
ρ_R	Beta	0.5000	0.2000
ρ_{zR}	Normal	0.0000	0.4000
$M_{r\zeta}$	Normal	0.0000	1.0000
$M_{z\zeta}$	Normal	0.0000	1.0000
$M_{R\zeta}$	Normal	0.0000	1.0000
σ_r	Inverse Gamma	0.2500	4.0000
σ_z	Inverse Gamma	0.8000	4.0000
σ_R	Inverse Gamma	0.3000	4.0000
σ_ζ	Inverse Gamma	0.2000	4.0000

Table 10: Parameter Estimation Results-Fixed Exchange Rate, Current-looking Monetary Policy, China

Parameter	Mean	Standard Deviation	90 % Posterior Interval Lower Bound	90 % Posterior Interval Upper Bound
a_π	0.2233	0.1079	0.0490	0.3929
a_x	0.2228	0.1304	0.0307	0.4100
ω	0.5238	0.0403	0.4569	0.5904
π^*	1.6333	0.4921	0.8176	2.4383
r^*	0.8588	0.3797	0.2562	1.4656
φ	3.5772	0.8794	2.1815	4.9473
ρ_z	0.9271	0.0464	0.8631	0.9991
ρ_R	0.6162	0.1577	0.3973	0.9181
ρ_{zR}	0.7459	0.0000	0.7459	0.7459
$M_{r\zeta}$	-0.4526	0.0000	-0.4526	-0.4526
$M_{z\zeta}$	0.3725	0.0000	0.3725	0.3725
$M_{R\zeta}$	-0.2618	0.0000	-0.2618	-0.2618
σ_r	0.1456	0.0211	0.1123	0.1787
σ_z	0.3814	0.0463	0.3074	0.4582
σ_R	0.3775	0.0722	0.2559	0.4898
σ_ζ	0.0986	0.0145	0.0751	0.1210

Notes: The posterior summary statistics are calculated by the Metropolis-Hastings algorithm.

Table 11: Determinacy versus Indeterminacy-Fixed Exchange Rate, Current-looking Monetary Policy, China

Probability	
Determinacy	Indeterminacy
0.0327	0.9673

Notes: The posterior probabilities are calculated by the Metropolis-Hastings algorithm.

Table 12: Variance Decomposition-Fixed Exchange Rate, Current-looking Monetary Policy, China

	Output Gap	Inflation	Interest Rate
Monetary Policy Shock	0.0592 [0.0047, 0.1398]	0.0025 [0.0000, 0.0056]	0.1339 [0.0000, 0.3496]
Technology Shock	0.4553 [0.3316, 0.5970]	0.6906 [0.6003, 0.8085]	0.5823 [0.3808, 0.7581]
Foreign Interest Rate Shock	0.4661 [0.2271, 0.6335]	0.2855 [0.1562, 0.4023]	0.2479 [0.0965, 0.3856]
Sunspot Shock	0.0195 [0.0024, 0.0419]	0.0214 [0.0036, 0.0416]	0.0359 [0.0043, 0.0723]

Notes: This table reports the posterior mean and 90% probability intervals.

Figure 5: Impulse Responses-Fixed Exchange Rate, Current-looking Monetary Policy, China

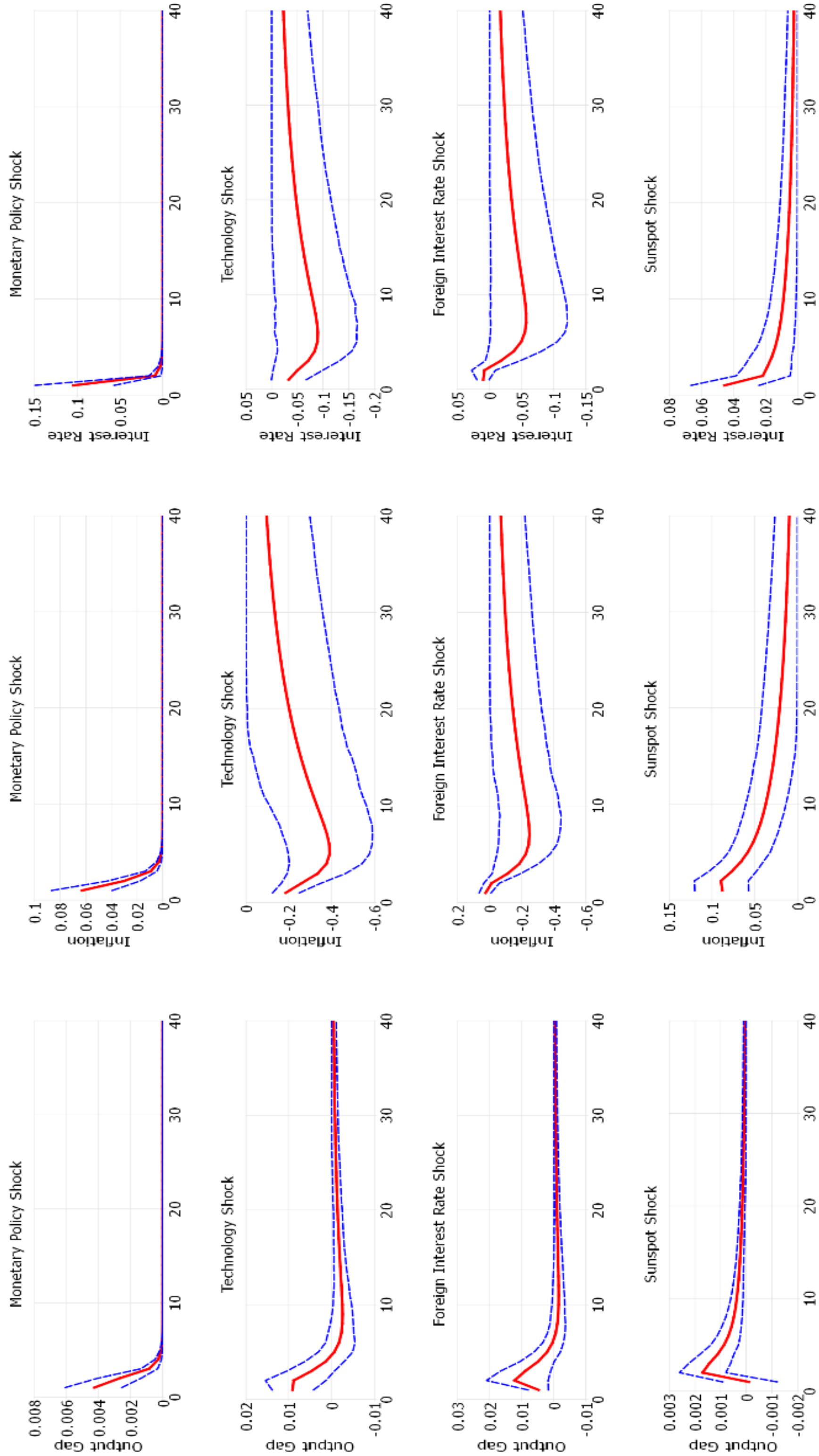


Table 13: Prior Distributions-Flexible Exchange Rate, Forward-looking Monetary Policy, China

Parameter	Density	Prior Mean	Prior Standard Deviation
a_π	Gamma	1.1000	0.5000
a_x	Gamma	0.2500	0.1500
ω	Beta	0.8500	0.1000
π^*	Gamma	4.0000	2.0000
r^*	Gamma	2.0000	1.0000
φ	Gamma	2.0000	0.7500
ρ_z	Beta	0.9000	0.1000
ρ_R	Beta	0.5000	0.2000
ρ_π	Beta	0.7000	0.1000
ρ_{zR}	Normal	0.0000	0.4000
$\rho_{z\pi}$	Normal	0.0000	0.4000
$\rho_{R\pi}$	Normal	0.0000	0.4000
$M_{r\zeta}$	Normal	0.0000	1.0000
$M_{z\zeta}$	Normal	0.0000	1.0000
$M_{R\zeta}$	Normal	0.0000	1.0000
$M_{\pi\zeta}$	Normal	0.0000	1.0000
σ_r	Inverse Gamma	0.2500	4.0000
σ_z	Inverse Gamma	0.8000	4.0000

σ_R	Inverse Gamma	0.3000	4.0000
σ_π	Inverse Gamma	0.3000	4.0000
σ_ζ	Inverse Gamma	0.2000	4.0000

Table 14: Parameter Estimation Results-Flexible Exchange Rate, Forward-looking Monetary Policy, China

Parameter	Mean	Standard Deviation	90 % Posterior Interval Lower Bound	90 % Posterior Interval Upper Bound
a_π	0.5865	0.0828	0.5391	0.6395
a_x	0.4208	0.0202	0.4063	0.4515
ω	1.0336	0.0021	1.0322	1.0362
π^*	4.7384	0.0081	4.7321	4.7513
r^*	0.4026	0.1295	0.2939	0.5384
φ	1.2647	0.0715	1.2209	1.3865
ρ_z	0.8178	0.0082	0.8017	0.8187
ρ_R	0.2214	0.0186	0.2061	0.2289
ρ_π	0.8800	0.0118	0.8645	0.8847
ρ_{zR}	0.5536	0.0555	0.4776	0.5935
$\rho_{z\pi}$	-0.9811	0.0207	-0.9883	-0.9745

$\rho_{R\pi}$	-0.3931	0.0507	-0.4219	-0.3351
$M_{r\zeta}$	-0.2957	0.1272	-0.3137	-0.0037
$M_{z\zeta}$	0.4469	0.1332	0.2856	0.5151
$M_{R\zeta}$	0.0437	0.1892	-0.1567	0.2854
$M_{\pi\zeta}$	-0.4836	0.1460	-0.5804	-0.3983
σ_r	0.4161	0.0154	0.4003	0.4249
σ_z	0.3233	0.0104	0.3213	0.3364
σ_R	0.2343	0.0141	0.2170	0.2439
σ_π	0.4184	0.0122	0.4020	0.4257
σ_ζ	0.1672	0.0093	0.1572	0.1745

Notes: The posterior summary statistics are calculated by the Metropolis-Hastings algorithm.

Table 15: Determinacy versus Indeterminacy-Flexible Exchange Rate, Forward-looking Monetary Policy, China

Probability	
Determinacy	Indeterminacy
0.6071	0.3929

Notes: The posterior probabilities are calculated by the Metropolis-Hastings algorithm.

Table 16: Variance Decomposition-Flexible Exchange Rate, Forward-looking Monetary Policy, China

	Output Gap	Inflation	Interest Rate
Monetary Policy Shock	0.1318 [0.1057, 0.1696]	0.0008 [0.0005, 0.0013]	0.7961 [0.7484, 0.8720]
Technology Shock	0.2698 [0.2500, 0.2907]	0.3268 [0.3111, 0.3327]	0.0600 [0.0387, 0.0746]
Foreign Interest Rate Shock	0.0569 [0.0229, 0.0850]	0.0111 [0.0066, 0.0150]	0.0106 [0.0068, 0.0144]
Foreign Inflation Shock	0.5110 [0.4641, 0.5476]	0.6315 [0.6138, 0.6439]	0.1153 [0.0734, 0.1502]
Sunspot Shock	0.0304 [0.0133, 0.0404]	0.0297 [0.0149, 0.0383]	0.0180 [0.0025, 0.0224]

Notes: This table reports the posterior mean and 90% probability intervals.

Figure 6: Impulse Responses-Flexible Exchange Rate, Forward-looking Monetary Policy, China

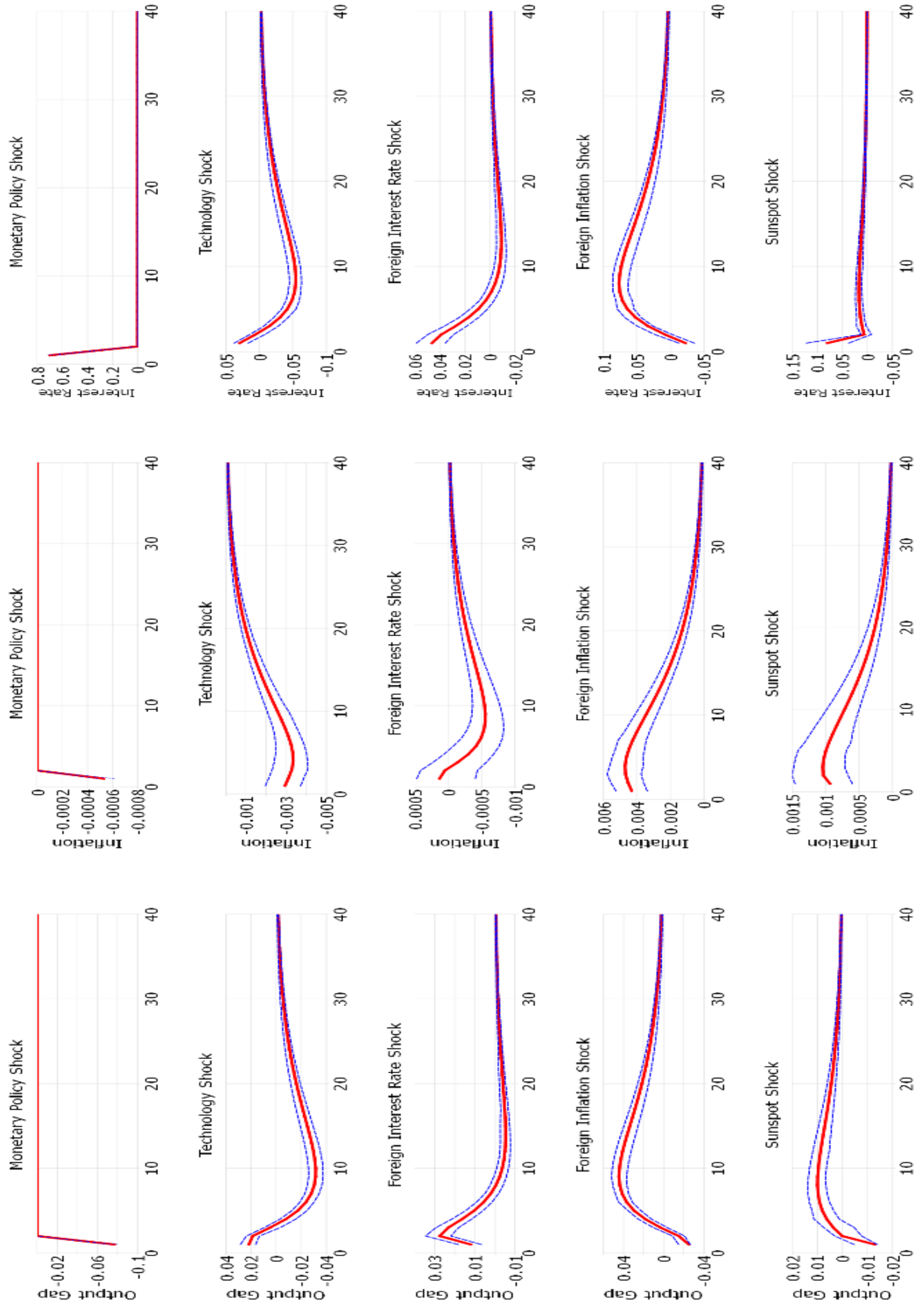


Table 17: Prior Distributions-Fixed Exchange Rate, Forward-looking Monetary Policy, China

Parameter	Density	Prior Mean	Prior Standard Deviation
a_π	Gamma	0.8000	0.5000
a_x	Gamma	0.2500	0.1500
ω	Beta	0.8000	0.1000
π^*	Gamma	4.0000	2.0000
r^*	Gamma	2.0000	1.0000
φ	Gamma	2.0000	0.7500
ρ_z	Beta	0.9000	0.1000
ρ_R	Beta	0.5000	0.2000
ρ_{zR}	Normal	0.0000	0.4000
$M_{r\zeta}$	Normal	0.0000	1.0000
$M_{z\zeta}$	Normal	0.0000	1.0000
$M_{R\zeta}$	Normal	0.0000	1.0000
σ_r	Inverse Gamma	0.2500	4.0000
σ_z	Inverse Gamma	0.8000	4.0000
σ_R	Inverse Gamma	0.3000	4.0000
σ_ζ	Inverse Gamma	0.2000	4.0000

Table 18: Parameter Estimation Results-Fixed Exchange Rate, Forward-looking Monetary Policy, China

Parameter	Mean	Standard Deviation	90 % Posterior Interval Lower Bound	90 % Posterior Interval Upper Bound
a_π	0.3053	0.1138	0.1186	0.4917
a_x	0.2931	0.1478	0.0563	0.5139
ω	0.6624	0.0291	0.6152	0.7109
π^*	1.5837	0.5183	0.7034	2.3970
r^*	0.7854	0.3560	0.2126	1.3447
φ	2.6055	0.7281	1.4279	3.7583
ρ_z	0.9423	0.0406	0.8882	0.9996
ρ_R	0.5040	0.1079	0.3299	0.6768
ρ_{zR}	0.2947	0.0000	0.2947	0.2947
$M_{r\zeta}$	-0.9480	0.0000	-0.9480	-0.9480
$M_{z\zeta}$	0.0991	0.0000	0.0991	0.0991
$M_{R\zeta}$	-0.2598	0.0000	-0.2598	-0.2598
σ_r	0.2272	0.0306	0.1781	0.2759
σ_z	0.3278	0.0452	0.2567	0.3984
σ_R	0.2148	0.0403	0.1499	0.2759
σ_ζ	0.1049	0.0145	0.0815	0.1277

Notes: The posterior summary statistics are calculated by the Metropolis-Hastings algorithm.

Table 19: Determinacy versus Indeterminacy-Fixed Exchange Rate, Forward-looking Monetary Policy, China

Probability	
Determinacy	Indeterminacy
0.0322	0.9678

Notes: The posterior probabilities are calculated by the Metropolis-Hastings algorithm.

Table 20: Variance Decomposition-Fixed Exchange Rate, Forward-looking Monetary Policy, China

	Output Gap	Inflation	Interest Rate
Monetary Policy Shock	0.2795 [0.0740, 0.4802]	0.0403 [0.0000, 0.0917]	0.2900 [0.0006, 0.6070]
Technology Shock	0.6176 [0.3737, 0.8654]	0.9180 [0.8408, 0.9804]	0.6268 [0.2484, 0.9672]
Foreign Interest Rate Shock	0.0540 [0.0162, 0.0877]	0.0281 [0.0096, 0.0452]	0.0206 [0.0039, 0.0368]
Sunspot Shock	0.0490 [0.0113, 0.0858]	0.0137 [0.0005, 0.0303]	0.0626 [0.0012, 0.1226]

Notes: This table reports the posterior mean and 90% probability intervals.

Figure 7: Impulse Responses-Fixed Exchange Rate, Forward-looking Monetary Policy, China

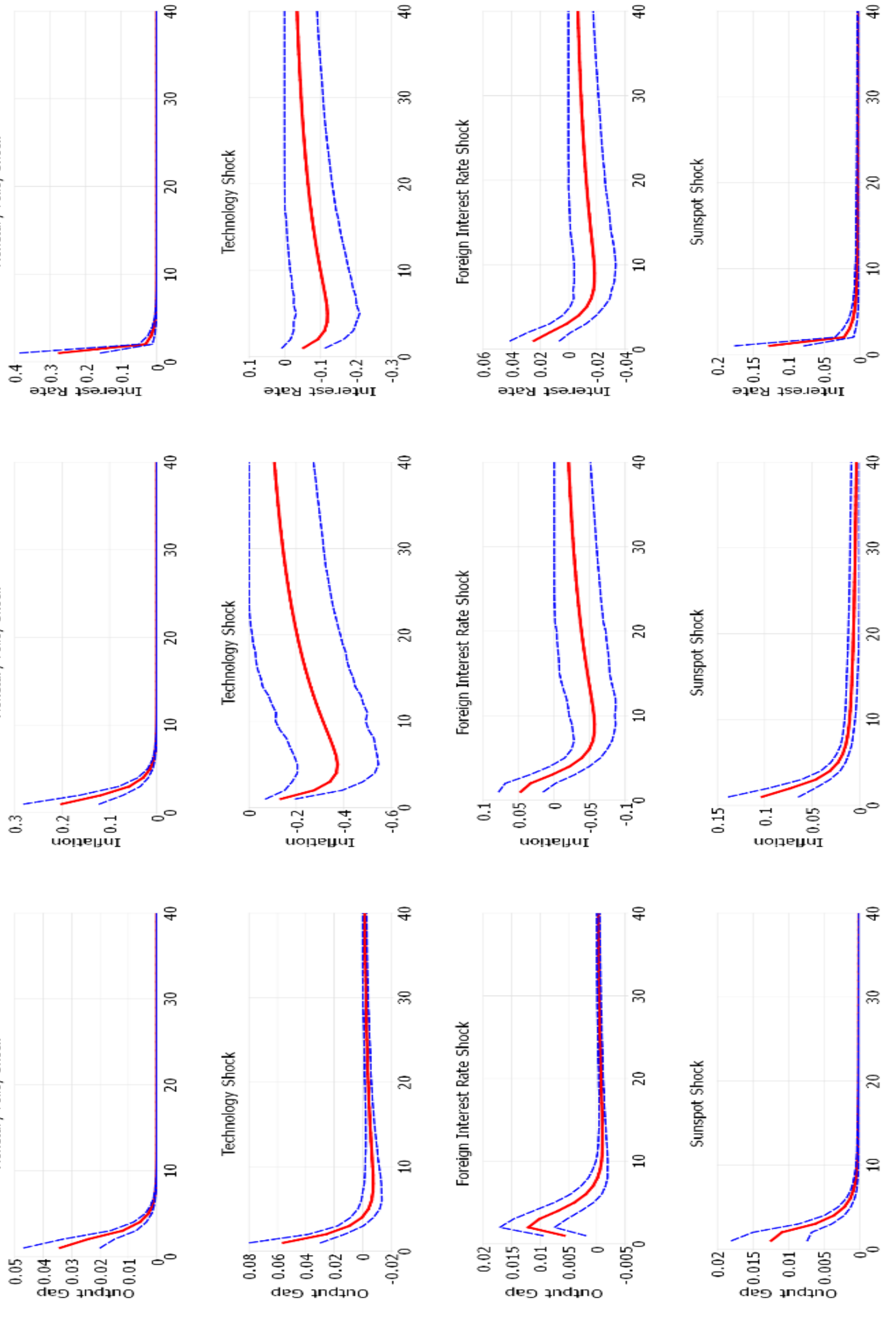


Table 21: Prior Distributions-Flexible Exchange Rate, Current-looking Monetary Policy, US

Parameter	Density	Prior Mean	Prior Standard Deviation
a_π	Gamma	1.1000	0.5000
a_x	Gamma	0.2500	0.1500
ω	Beta	0.8000	0.1000
π^*	Gamma	4.0000	2.0000
r^*	Gamma	2.0000	1.0000
φ	Gamma	2.0000	0.7500
ρ_z	Beta	0.9000	0.1000
ρ_R	Beta	0.5000	0.2000
ρ_π	Beta	0.7000	0.1000
ρ_{zR}	Normal	0.0000	0.4000
$\rho_{z\pi}$	Normal	0.0000	0.4000
$\rho_{R\pi}$	Normal	0.0000	0.4000
$M_{r\zeta}$	Normal	0.0000	1.0000
$M_{z\zeta}$	Normal	0.0000	1.0000
$M_{R\zeta}$	Normal	0.0000	1.0000
$M_{\pi\zeta}$	Normal	0.0000	1.0000
σ_r	Inverse Gamma	0.2500	4.0000
σ_z	Inverse Gamma	0.8000	4.0000

σ_R	Inverse Gamma	0.3000	4.0000
σ_π	Inverse Gamma	0.3000	4.0000
σ_ζ	Inverse Gamma	0.2000	4.0000

Table 22: Parameter Estimation Results-Flexible Exchange Rate, Current-looking Monetary Policy, US

Parameter	Mean	Standard Deviation	90 % Posterior Interval Lower Bound	90 % Posterior Interval Upper Bound
a_π	6.1792	0.7510	4.9870	7.4904
a_x	0.2792	0.1573	0.0409	0.5139
ω	0.3719	0.0395	0.3138	0.4360
π^*	1.7728	0.1161	1.6309	1.9677
r^*	1.2393	0.6070	0.5059	2.0725
φ	2.7482	0.3937	2.1763	3.3829
ρ_z	0.9156	0.0152	0.8855	0.9390
ρ_R	0.8957	0.0215	0.8598	0.9315
ρ_π	0.9099	0.0159	0.8850	0.9412
ρ_{zR}	0.6171	0.0000	0.6171	0.6171
$\rho_{z\pi}$	-0.7198	0.0000	-0.7198	-0.7198

$\rho_{R\pi}$	0.1020	0.0000	0.1020	0.1020
$M_{r\zeta}$	0.0354	0.4684	-0.5633	0.9368
$M_{z\zeta}$	0.0000	0.0000	0.0000	0.0000
$M_{R\zeta}$	0.0000	0.0000	0.0000	0.0000
$M_{\pi\zeta}$	0.0000	0.0003	-0.0004	0.0006
σ_r	0.1624	0.0261	0.1218	0.1946
σ_z	0.3721	0.0501	0.2795	0.4289
σ_R	0.3818	0.0378	0.3190	0.4345
σ_π	0.3856	0.0500	0.3030	0.4555
σ_ζ	0.2034	0.0597	0.1216	0.3116

Notes: The posterior summary statistics are calculated by the Metropolis-Hastings algorithm.

Table 23: Determinacy versus Indeterminacy-Flexible Exchange Rate, Current-looking Monetary Policy, US

Probability	
Determinacy	Indeterminacy
0.5724	0.4276

Notes: The posterior probabilities are calculated by the Metropolis-Hastings algorithm.

Table 24: Variance Decomposition-Flexible Exchange Rate, Current-looking Monetary Policy, US

	Output Gap	Inflation	Interest Rate
Monetary Policy Shock	0.3717 [0.1072, 0.5627]	0.0189 [0.0005, 0.0382]	0.0006 [0.0000, 0.0014]
Technology Shock	0.1543 [0.0119, 0.3653]	0.3822 [0.2511, 0.4990]	0.3911 [0.2620, 0.4999]
Foreign Interest Rate Shock	0.2979 [0.1074, 0.4873]	0.3372 [0.1108, 0.5652]	0.3475 [0.1112, 0.5788]
Foreign Inflation Shock	0.0636 [0.0097, 0.1425]	0.2545 [0.0796, 0.4554]	0.2606 [0.0837, 0.4629]
Sunspot Shock	0.1125 [0.0000, 0.2748]	0.0071 [0.0000, 0.0215]	0.0002 [0.0000, 0.0005]

Notes: This table reports the posterior mean and 90% probability intervals.

Figure 8: Impulse Responses-Flexible Exchange Rate, Current-looking Monetary Policy, US

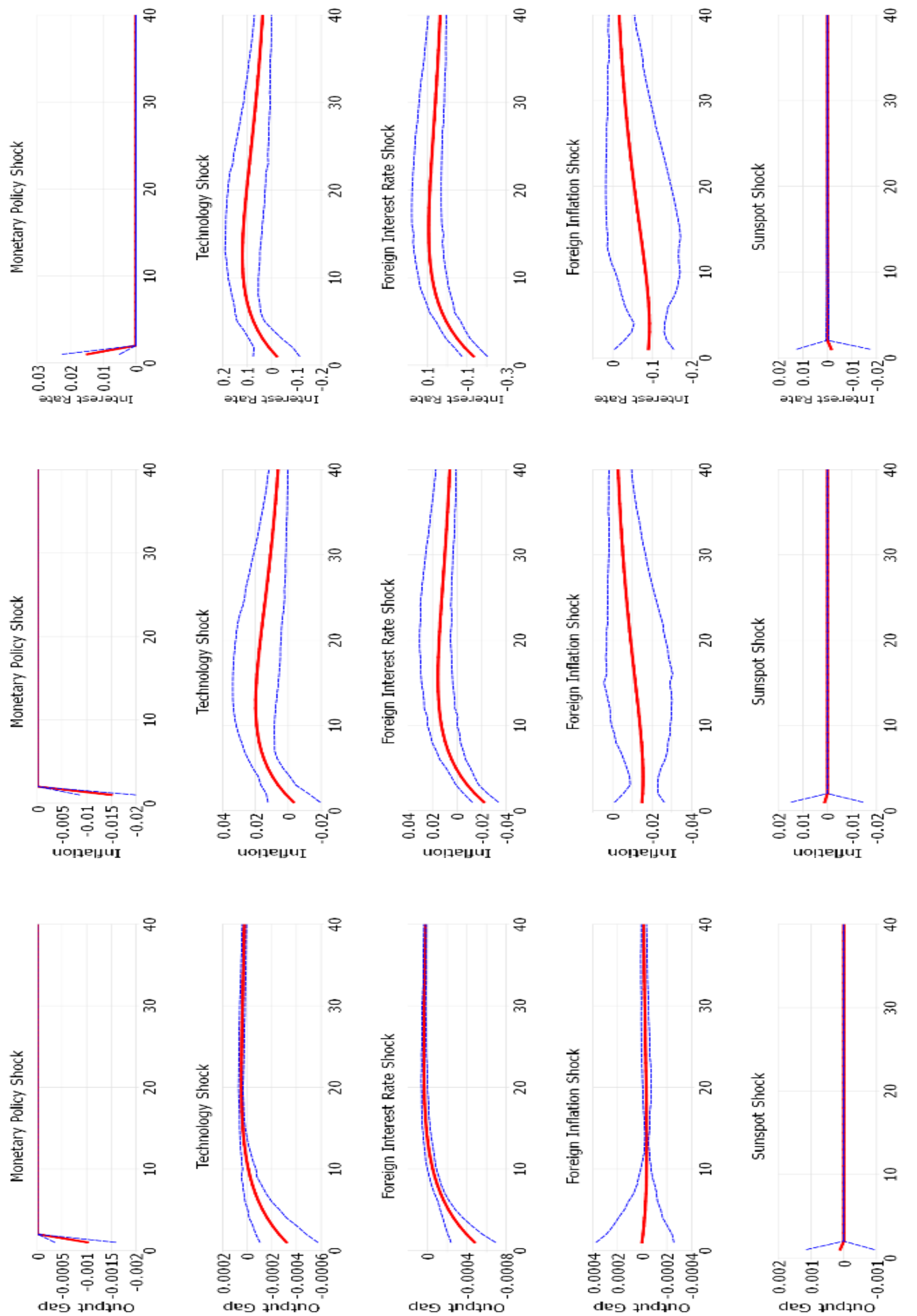


Table 25: Prior Distributions-Fixed Exchange Rate, Current-looking Monetary Policy, US

Parameter	Density	Prior Mean	Prior Standard Deviation
a_π	Gamma	0.8000	0.5000
a_x	Gamma	0.2500	0.1500
ω	Beta	0.8000	0.1000
π^*	Gamma	4.0000	2.0000
r^*	Gamma	2.0000	1.0000
φ	Gamma	2.0000	0.7500
ρ_z	Beta	0.9000	0.1000
ρ_R	Beta	0.5000	0.2000
ρ_{zR}	Normal	0.0000	0.4000
$M_{r\zeta}$	Normal	0.0000	1.0000
$M_{z\zeta}$	Normal	0.0000	1.0000
$M_{R\zeta}$	Normal	0.0000	1.0000
σ_r	Inverse Gamma	0.2500	4.0000
σ_z	Inverse Gamma	0.8000	4.0000
σ_R	Inverse Gamma	0.3000	4.0000
σ_ζ	Inverse Gamma	0.2000	4.0000

Table 26: Parameter Estimation Results-Fixed Exchange Rate, Current-looking Monetary Policy, US

Parameter	Mean	Standard Deviation	90 % Posterior Interval Lower Bound	90 % Posterior Interval Upper Bound
a_π	6.9062	0.3601	6.3201	7.3460
a_x	0.3320	0.1708	0.1109	0.6141
ω	0.2334	0.0209	0.1987	0.2641
π^*	1.3130	0.0096	1.2985	1.3303
r^*	0.4458	0.0529	0.3543	0.5315
φ	2.1193	0.2347	1.8307	2.5244
ρ_z	0.7233	0.0220	0.6915	0.7555
ρ_R	0.7051	0.0241	0.6699	0.7420
ρ_{zR}	0.9994	0.0005	0.9991	0.9998
$M_{r\zeta}$	-0.0775	0.4253	-0.8105	0.6001
$M_{z\zeta}$	0．0767	0.3540	-0.4332	0.6998
$M_{R\zeta}$	0.0763	0.3531	-0.4364	0.6973
σ_r	0.1134	0.0114	0.0988	0.1358
σ_z	0.4786	0.0743	0.3796	0.5921
σ_R	0.7335	0.0816	0.6235	0.8522
σ_ζ	0.3011	0.1147	0.1442	0.4671

Notes: The posterior summary statistics are calculated by the Metropolis-Hastings algorithm.

Table 27: Determinacy versus Indeterminacy-Fixed Exchange Rate, Current-looking Monetary Policy, US

Probability	
Determinacy	Indeterminacy
0.0317	0.9683

Notes: The posterior probabilities are calculated by the Metropolis-Hastings algorithm.

Table 28: Variance Decomposition-Fixed Exchange Rate, Current-looking Monetary Policy, US

	Output Gap	Inflation	Interest Rate
Monetary Policy Shock	0.0424 [0.0139, 0.0666]	0.0155 [0.0042, 0.0253]	0.0001 [0.0000, 0.0001]
Technology Shock	0.2629 [0.2293, 0.3029]	0.2773 [0.2501, 0.3125]	0.2863 [0.2641, 0.3221]
Foreign Interest Rate Shock	0.6352 [0.5530, 0.7004]	0.6583 [0.5985, 0.7074]	0.6797 [0.6250, 0.7191]
Sunspot Shock	0.0594 [0.0002, 0.1664]	0.0489 [0.0001, 0.1098]	0.0339 [0.0000, 0.0766]

Notes: This table reports the posterior mean and 90% probability intervals.

Figure 9: Impulse Responses-Fixed Exchange Rate, Current-looking Monetary Policy, US

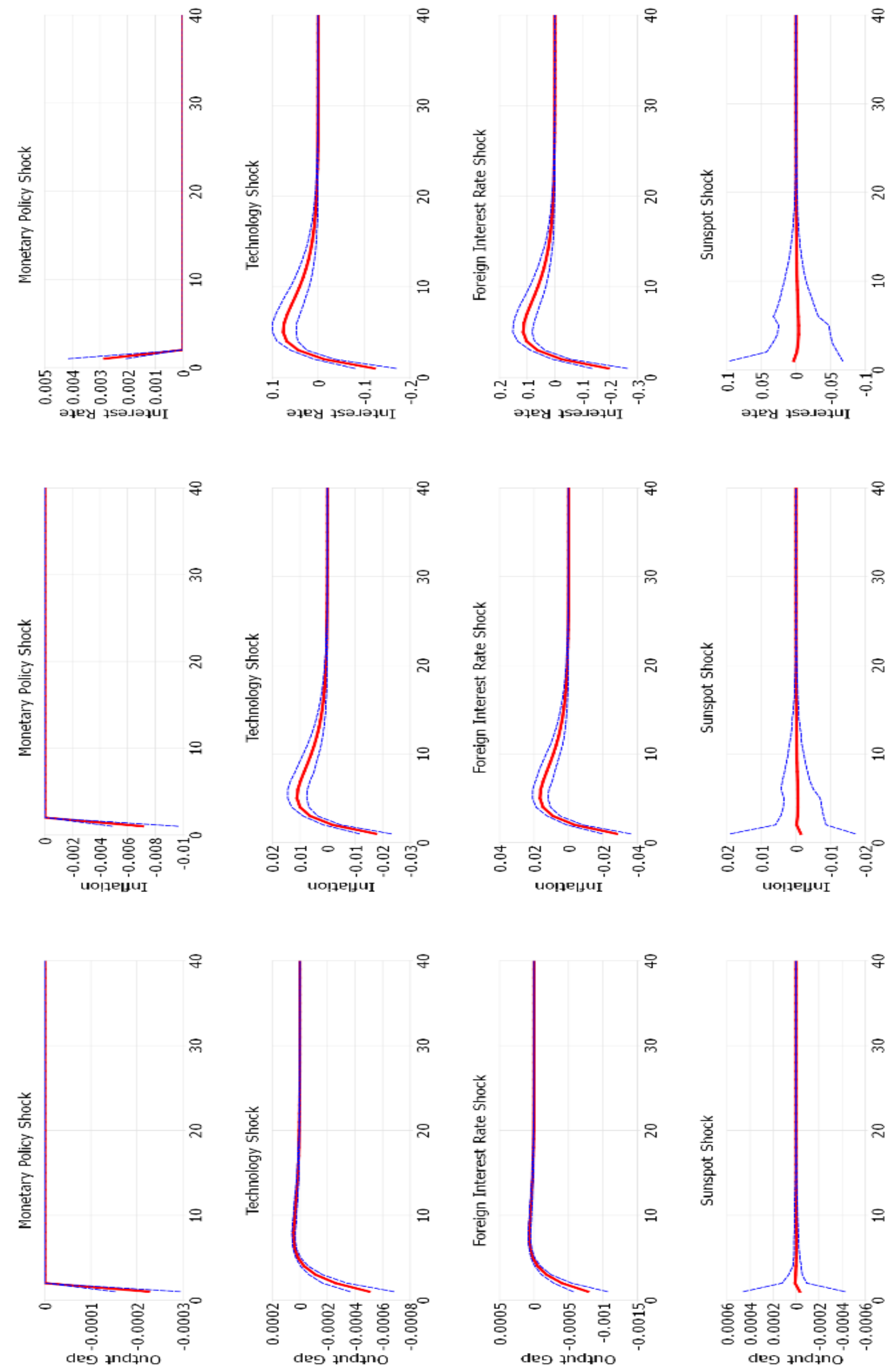


Table 29: Prior Distributions-Flexible Exchange Rate, Forward-looking Monetary Policy, US

Parameter	Density	Prior Mean	Prior Standard Deviation
a_π	Gamma	1.1000	0.5000
a_x	Gamma	0.2500	0.1500
ω	Beta	0.8500	0.1000
π^*	Gamma	4.0000	2.0000
r^*	Gamma	2.0000	1.0000
φ	Gamma	2.0000	0.7500
ρ_z	Beta	0.9000	0.1000
ρ_R	Beta	0.5000	0.2000
ρ_π	Beta	0.7000	0.1000
ρ_{zR}	Normal	0.0000	0.4000
$\rho_{z\pi}$	Normal	0.0000	0.4000
$\rho_{R\pi}$	Normal	0.0000	0.4000
$M_{r\zeta}$	Normal	0.0000	1.0000
$M_{z\zeta}$	Normal	0.0000	1.0000
$M_{R\zeta}$	Normal	0.0000	1.0000
$M_{\pi\zeta}$	Normal	0.0000	1.0000
σ_r	Inverse Gamma	0.2500	4.0000
σ_z	Inverse Gamma	0.8000	4.0000

σ_R	Inverse Gamma	0.3000	4.0000
σ_π	Inverse Gamma	0.3000	4.0000
σ_ζ	Inverse Gamma	0.2000	4.0000

Table 30: Parameter Estimation Results-Flexible Exchange Rate, Forward-looking Monetary Policy, US

Parameter	Mean	Standard Deviation	90 % Posterior Interval Lower Bound	90 % Posterior Interval Upper Bound
a_π	1.6570	1.0416	0.1924	2.4429
a_x	0.4131	0.0554	0.3542	0.5074
ω	0.9640	0.0041	0.9560	0.9682
π^*	1.5950	0.0036	1.5884	1.5997
r^*	0.5024	0.0798	0.3780	0.6295
φ	0.8224	0.1651	0.5538	1.0051
ρ_z	0.8792	0.0161	0.8624	0.9046
ρ_R	0.8897	0.0013	0.8874	0.8913
ρ_π	0.8577	0.0077	0.8467	0.8677

ρ_{zR}	0.5246	0.0000	0.5246	0.5246
$\rho_{z\pi}$	-0.6810	0.0000	-0.6810	-0.6810
$\rho_{R\pi}$	0.2662	0.0000	0.2662	0.2662
$M_{r\zeta}$	-0.0669	0.2944	-0.4287	0.4202
$M_{z\zeta}$	0.0000	0.0000	0.0000	0.0000
$M_{R\zeta}$	0.0000	0.0000	0.0000	0.0000
$M_{\pi\zeta}$	0.0000	0.0002	-0.0003	0.0002
σ_r	0.1017	0.0050	0.0952	0.1113
σ_z	0.2072	0.0382	0.1662	0.2516
σ_R	0.2725	0.0374	0.2241	0.3192
σ_π	0.3018	0.0409	0.2558	0.3646
σ_ζ	0.2618	0.0253	0.2234	0.2990

Notes: The posterior summary statistics are calculated by the Metropolis-Hastings algorithm.

Table 31: Determinacy versus Indeterminacy-Flexible Exchange Rate, Forward-looking Monetary Policy, US

Probability	
Determinacy	Indeterminacy
0.6068	0.3932

Notes: The posterior probabilities are calculated by the Metropolis-Hastings algorithm.

Table 32: Variance Decomposition-Flexible Exchange Rate, Forward-looking Monetary Policy, US

	Output Gap	Inflation	Interest Rate
Monetary Policy Shock	0.0081 [0.0001, 0.0276]	0.0001 [0.0000, 0.0003]	0.0825 [0.0026, 0.2231]
Technology Shock	0.2662 [0.0879, 0.5720]	0.1968 [0.0055, 0.5188]	0.2310 [0.0192, 0.5198]
Foreign Interest Rate Shock	0.1505 [0.0058, 0.3225]	0.2782 [0.0014, 0.6151]	0.1391 [0.0040, 0.2815]
Foreign Inflation Shock	0.5716 [0.3758, 0.7391]	0.5249 [0.3662, 0.7795]	0.5140 [0.3588, 0.7001]
Sunspot Shock	0.0036 [0.0000, 0.0079]	0.0000 [0.0000, 0.0001]	0.0334 [0.0000, 0.0795]

Notes: This table reports the posterior mean and 90% probability intervals.

Figure 10: Impulse Responses-Flexible Exchange Rate, Forward-looking Monetary Policy, US

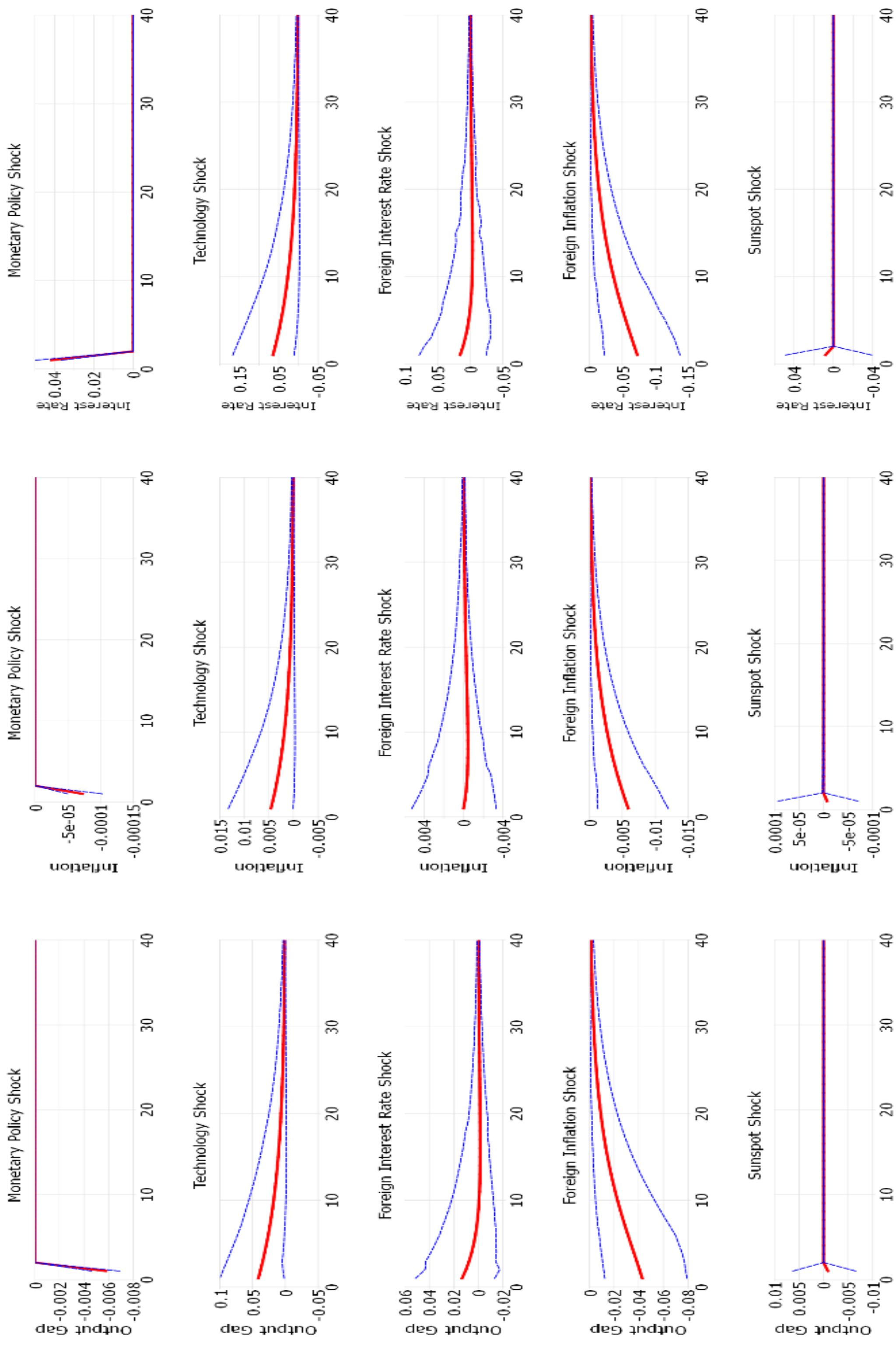


Table 33: Prior Distributions-Fixed Exchange Rate, Forward-looking Monetary Policy, US

Parameter	Density	Prior Mean	Prior Standard Deviation
a_π	Gamma	0.8000	0.5000
a_x	Gamma	0.2500	0.1500
ω	Beta	0.8000	0.1000
π^*	Gamma	4.0000	2.0000
r^*	Gamma	2.0000	1.0000
φ	Gamma	2.0000	0.7500
ρ_z	Beta	0.9000	0.1000
ρ_R	Beta	0.5000	0.2000
ρ_{zR}	Normal	0.0000	0.4000
$M_{r\zeta}$	Normal	0.0000	1.0000
$M_{z\zeta}$	Normal	0.0000	1.0000
$M_{R\zeta}$	Normal	0.0000	1.0000
σ_r	Inverse Gamma	0.2500	4.0000
σ_z	Inverse Gamma	0.8000	4.0000
σ_R	Inverse Gamma	0.3000	4.0000
σ_ζ	Inverse Gamma	0.2000	4.0000

Table 34: Parameter Estimation Results-Fixed Exchange Rate, Forward-looking Monetary Policy, US

Parameter	Mean	Standard Deviation	90 % Posterior Interval Lower Bound	90 % Posterior Interval Upper Bound
a_π	1.2520	0.0437	1.2025	1.3038
a_x	0.3807	0.0449	0.3228	0.4413
ω	0.1430	0.0077	0.1327	0.1530
π^*	2.8632	0.0369	2.8170	2.9097
r^*	0.7741	0.0526	0.6957	0.8374
φ	3.8277	0.0000	3.8277	3.8277
ρ_z	0.8595	0.0635	0.7984	0.9419
ρ_R	0.7576	0.0979	0.6392	0.8794
ρ_{zR}	0.9823	0.0118	0.9726	0.9904
$M_{r\zeta}$	-0.6621	0.0468	-0.7368	-0.6074
$M_{z\zeta}$	0.0192	0.1281	-0.1577	0.1252
$M_{R\zeta}$	-0.1151	0.1419	-0.3249	0.0083
σ_r	0.1093	0.0030	0.1045	0.1116
σ_z	0.1791	0.0022	0.1752	0.1814
σ_R	0.2200	0.0148	0.2027	0.2418
σ_ζ	0.0455	0.0015	0.0435	0.0475

Notes: The posterior summary statistics are calculated by the Metropolis-Hastings algorithm.

Table 35: Determinacy versus Indeterminacy-Fixed Exchange Rate, Forward-looking Monetary Policy, US

Probability	
Determinacy	Indeterminacy
0.0315	0.9685

Notes: The posterior probabilities are calculated by the Metropolis-Hastings algorithm.

Table 36: Variance Decomposition-Fixed Exchange Rate, Forward-looking Monetary Policy, US

	Output Gap	Inflation	Interest Rate
Monetary Policy Shock	0.2991 [0.0546, 0.5474]	0.0510 [0.0131, 0.1008]	0.0728 [0.0252, 0.1305]
Technology Shock	0.2629 [0.1600, 0.3538]	0.3978 [0.3690, 0.4216]	0.3897 [0.3772, 0.4374]
Foreign Interest Rate Shock	0.4138 [0.2512, 0.5847]	0.5400 [0.4825, 0.6052]	0.5250 [0.4669, 0.5886]
Sunspot Shock	0.0242 [0.0068, 0.0413]	0.0111 [0.0053, 0.0191]	0.0125 [0.0059, 0.0214]

Notes: This table reports the posterior mean and 90% probability intervals.

Figure 11: Impulse Responses-Fixed Exchange Rate, Forward-looking Monetary Policy, US

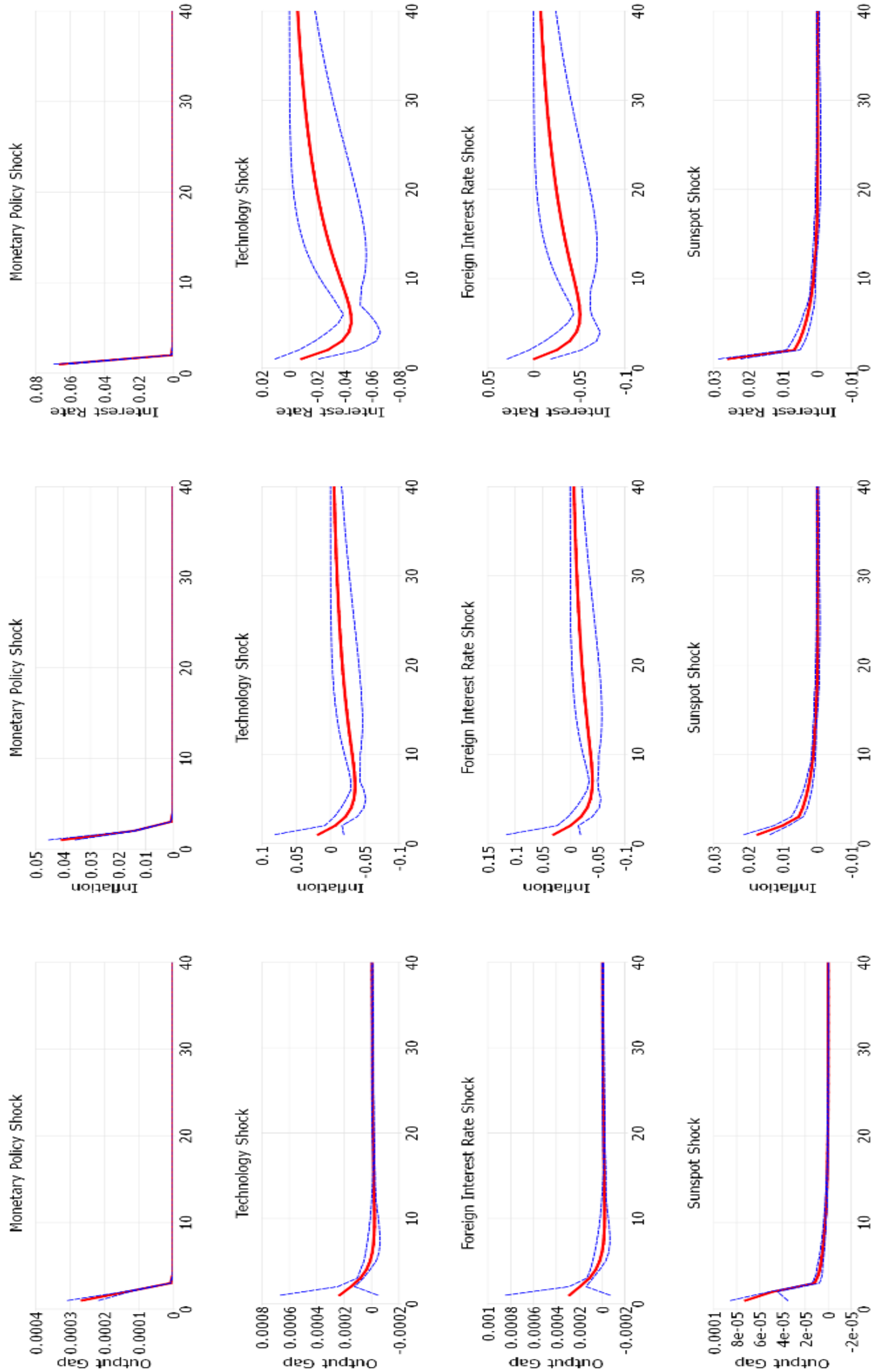


Table 37: Numerical Bifurcation Results

Variable parameter	Fixed point continuation	Eigenvalues	Origin	Bifurcation continuation	
Vary a	(1) Branch point a = 4.88, b = 3.88	Real and positive	Unstable improper node	Backward	Branch point
	(2) Period doubling a = -4.88, b = 3.88	Real and negative	Asymptotically stable improper node	Forward	Resonance 1-2 LPPD
Vary b	(3) Branch point a = 4.85, b = 3.85	Real and positive	Unstable improper node	Backward	Branch point
	(4) Neutral saddle a = 4.85, b = 1	Real and positive	Unstable improper node		
	(5) Period doubling a = 4.85, b = -5.85	Real with opposite signs	Saddle point	Backward	LPPD Resonance 1-2

Figure 12: Branch point (a = 4.88, b = 3.88)

eigenvalues	trace	determinant	Δ
{3.88, 1.00}	4.88	3.88	8.29

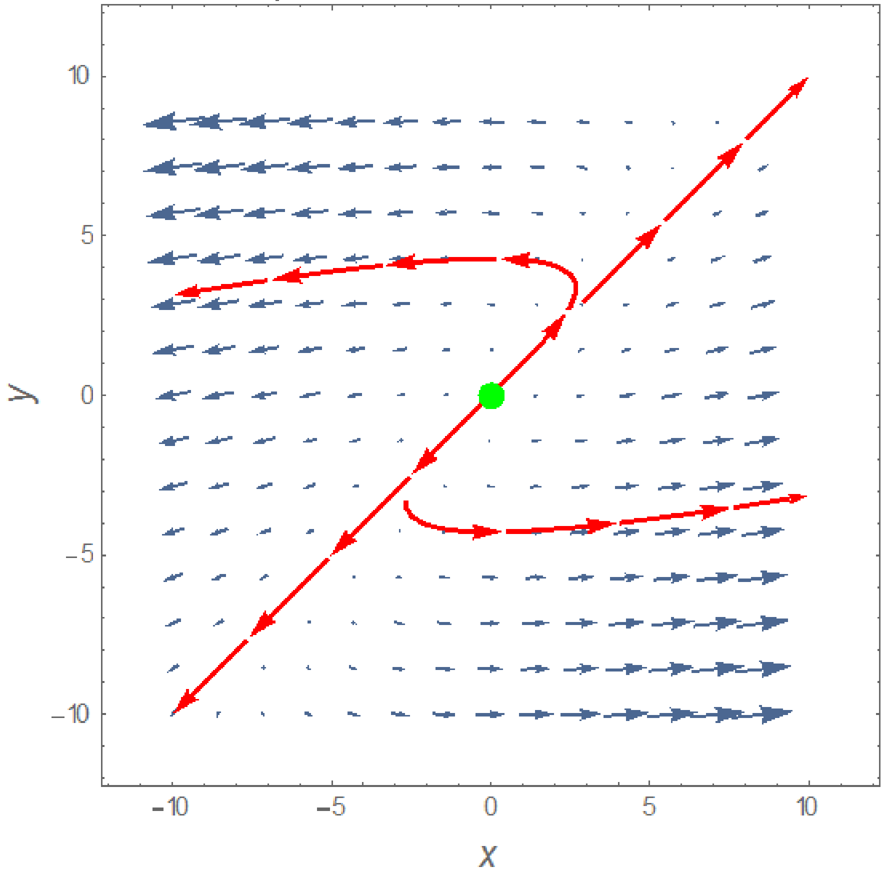


Figure 13: Period doubling (a = -4.88, b = 3.88)

eigenvalues	trace	determinant	Δ
{−3.88, −1.00}	−4.88	3.88	8.29

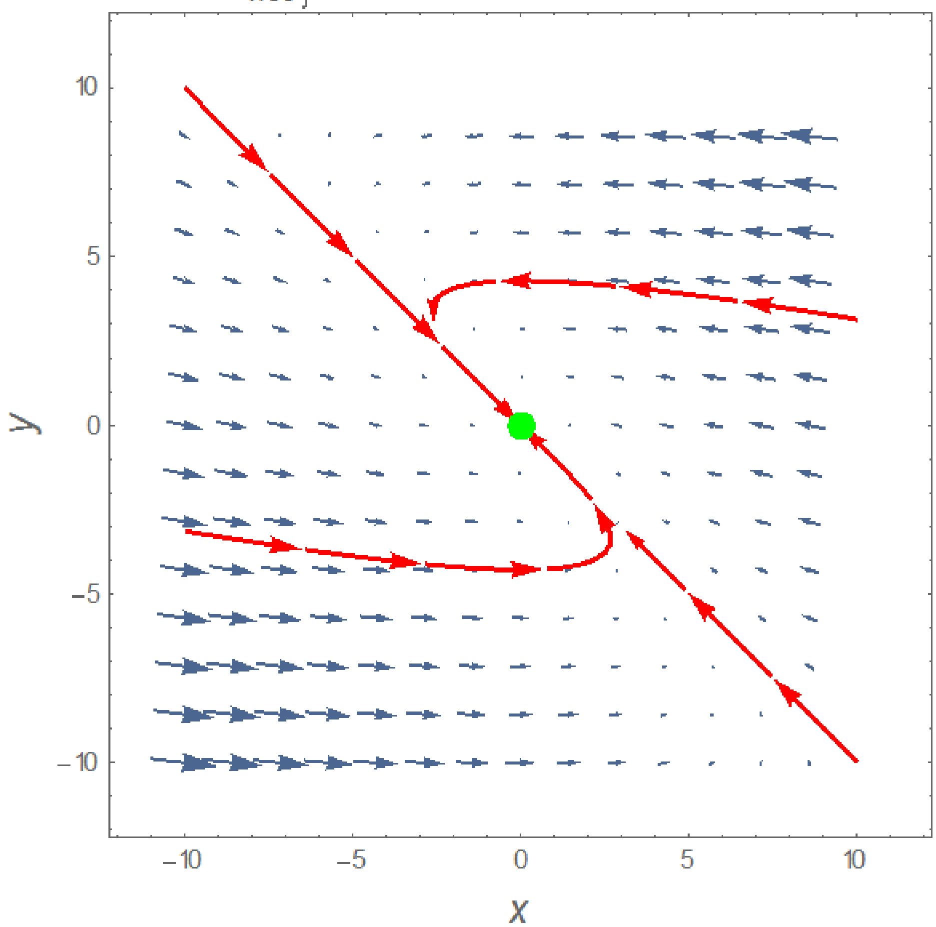


Figure 14: Branch point (a = 4.85, b = 3.85)

eigenvalues	trace	determinant	Δ
{3.85, 1.00}	4.85	3.85	8.12

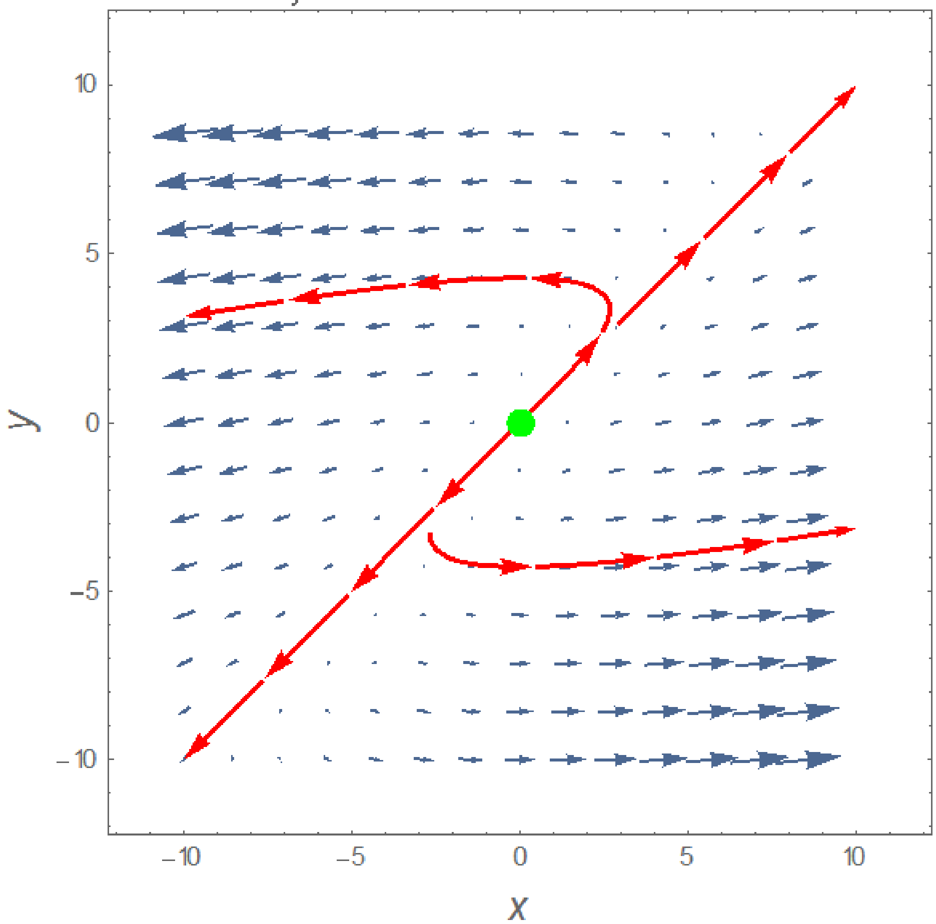


Figure 15: Neutral saddle (a = 4.85, b = 1)

eigenvalues	trace	determinant	Δ
{4.63, 0.22}	4.85	1.00	19.52

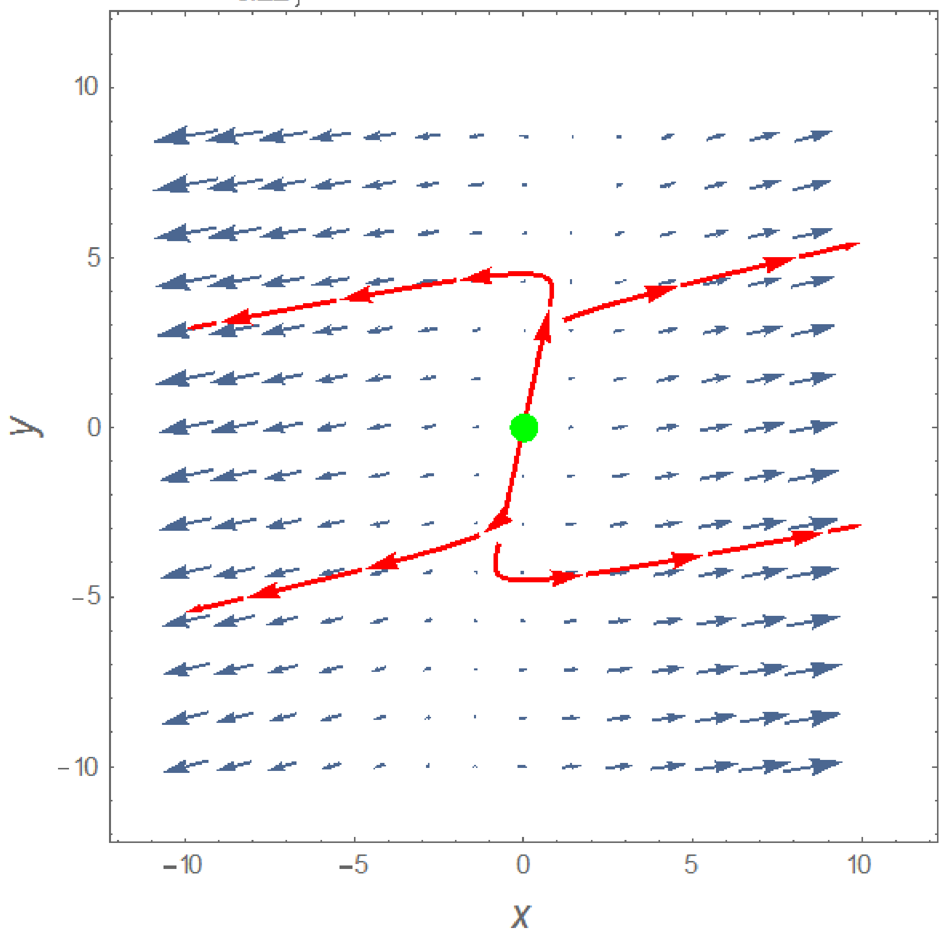


Figure 16: Period doubling (a = 4.85, b = -5.85)

eigenvalues	trace	determinant	Δ
{5.85, −1.00}	4.85	−5.85	46.92

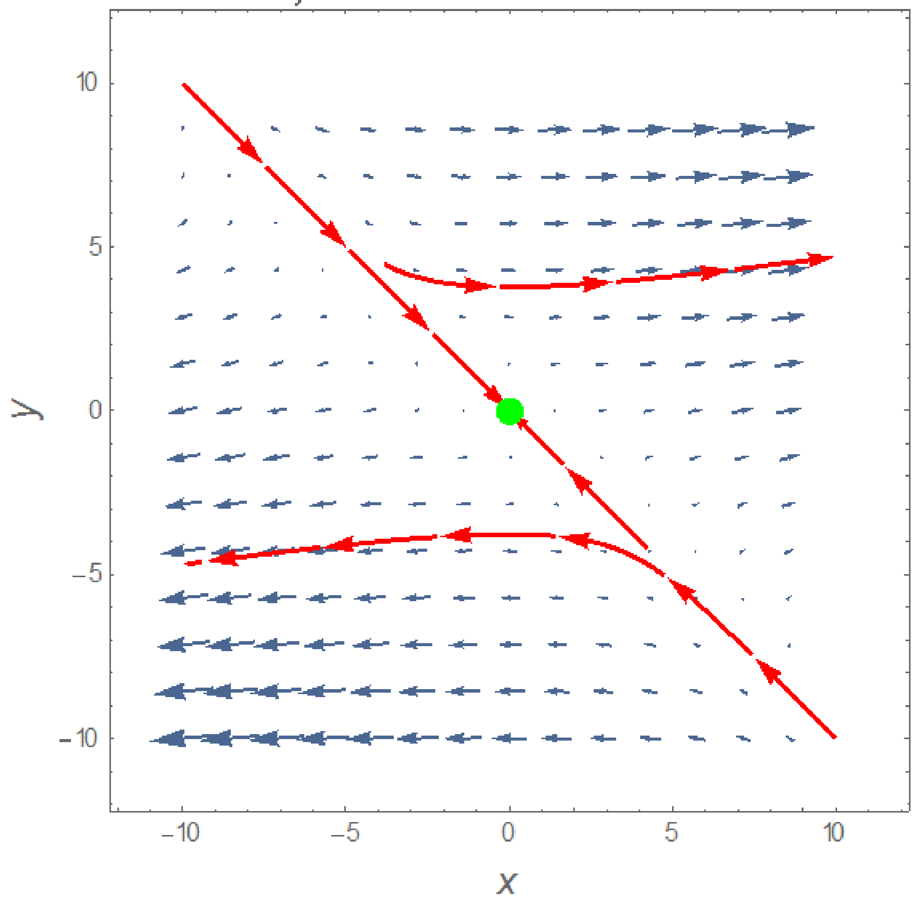
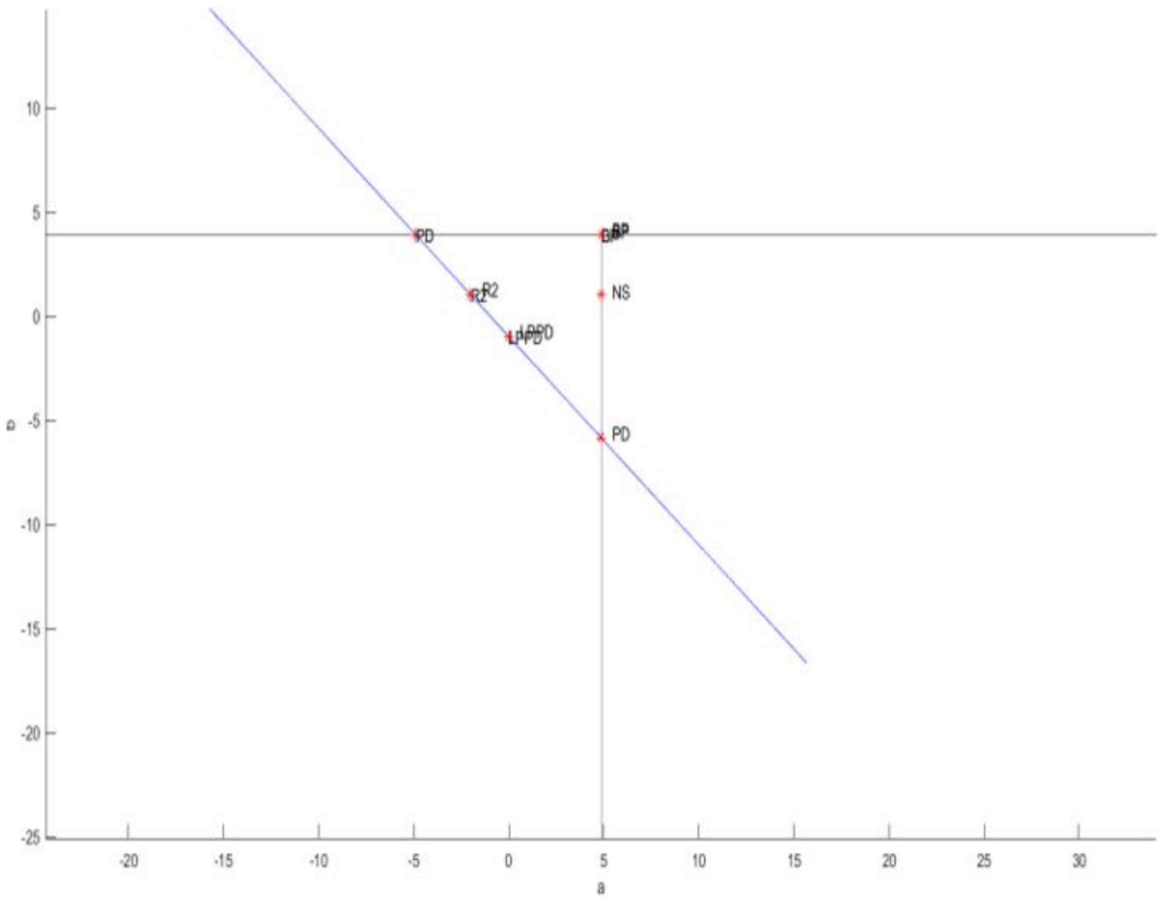


Figure 17: Bifurcation Curve in the (a, b)-Plane



1.11. Appendices

Appendix 1 Households Problem

A representative household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right],$$

where N_t denotes hours of labor, C_t is a composite consumption index defined by

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t} \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

with $C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}, C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$.

The household's budget constraint takes the form

$$\begin{aligned}
&\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t \left\{ Q_{t,t+1} D_{t+1} \right\} + \int_0^1 E_t \left\{ \mathcal{E}_{i,t} Q_{t,t+1}^i D_{t+1}^i \right\} di \\
&\leq W_t N_t + T_t + D_t + \int_0^1 \left(\frac{1+\tau_t}{1+\tau_t^i} \right) \mathcal{E}_{i,t} D_t^i di,
\end{aligned}$$

The optimal allocation of any given expenditure within each category of goods yields the demand functions, $C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}$ and $C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t}$,

where $P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ and $P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$.

So

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj = P_{H,t}C_{H,t} \text{ and } \int_0^1 P_{i,t}(j)C_{i,t}(j)dj = P_{i,t}C_{i,t}.$$

The optimal allocation of expenditures on imported goods by country of origin implies

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t}, \text{ where } P_{F,t} \equiv \left(\int_0^1 P_{i,t}{}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}},$$

so that

$$\int_0^1 P_{i,t}C_{i,t}di = P_{F,t}C_{F,t}.$$

The optimal allocation of expenditures between domestic and imported goods is given by

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \text{ and } C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \text{where}$$

$$P_t \equiv \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

so that

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t.$$

The effective nominal exchange rate is defined by $\mathcal{E}_t = \dfrac{\int_0^1 \mathcal{E}_{i,t}D_t^i di}{\int_0^1 D_t^i di}$. Hence

we have $\int_0^1 \mathcal{E}_{i,t}D_t^i di = \mathcal{E}_t \int_0^1 D_t^i di = \mathcal{E}_t D_t^*$ and

$$\int_0^1 \mathcal{E}_{i,t}Q_{t,t+1}^i D_{t+1}^i di = \int_0^1 \mathcal{E}_t Q_{t,t+1}^i D_{t+1}^* di = \mathcal{E}_t D_{t+1}^* \int_0^1 Q_{t,t+1}^i di = \mathcal{E}_t D_{t+1}^* Q_{t,t+1}^*.$$

Thus the budget constraint can be rewritten as

$$P_t C_t + E_t\left\{Q_{t,t+1} D_{t+1}\right\} + E_t\left\{\mathcal{E}_t Q^*_{t,t+1} D^*_{t+1}\right\} \le W_t N_t + T_t + D_t + (1+\tau_t)\mathcal{E}_t D^*_t \ .$$

Maximizing utility of a household subject to its budget constraint yields two Euler equations:

$$\beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{1}{Q_{t,t+1}}\right)\right\} = 1,$$
$$\beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)(1+\tau_{t+1})\left(\frac{1}{Q^*_{t,t+1}}\right)\right\} = 1.$$

The log-linearized form is

$$c_t = E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left(r_t - E_t\left\{\pi_{t+1}\right\} - \rho\right),$$
$$c_t = E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left(r^*_t + \left[E_t\left\{e_{t+1}\right\} - e_t\right] + E_t\left\{\tau_{t+1}\right\} - E_t\left\{\pi_{t+1}\right\} - \rho\right),$$

where $\left(R_t\right)^{-1} = E_t\left\{Q_{t,t+1}\right\}$ and $\left(R^*_t\right)^{-1} = E_t\left\{Q^*_{t,t+1}\right\}$ and

$$\pi_{t+1} \equiv p_{t+1} - p_t \equiv \log P_{t+1} - \log P_t \ .$$

Appendix 2 Backus-Smith Condition

Combined the Euler equations for the home country and country i, we get

$$\frac{Q^{*}_{t,t+1}}{Q_{t,t+1}} = \frac{\mathcal{E}_{t+1}}{\mathcal{E}_{t}}(1+\tau_{t+1}),$$
$$\frac{Q^{i*}_{t,t+1}}{Q^{i}_{t,t+1}} = \frac{\mathcal{E}_{i,t}}{\mathcal{E}_{i,t+1}},$$

$$\left(\frac{C_{t+1}}{C_t}\right) = \left(\frac{C^i_{t+1}}{C^i_t}\right)\left[\frac{\mathcal{Q}_{i,t+1}(1+\tau_{t+1})}{\mathcal{Q}_{i,t}}\right]^{\frac{1}{\sigma}} = \left(\frac{C^i_{t+1}}{C^i_t}\right)\left[\frac{\mathcal{Q}_{i,t+1}\Delta_{t+1}}{\mathcal{Q}_{i,t}\Delta_t}\right]^{\frac{1}{\sigma}},$$

where we define Δ and Θ to be the variables that captures the dynamics of τ_t, such that

$$1+\tau_{t+1} \equiv \frac{\Delta_{t+1}}{\Delta_t} \equiv \frac{\Theta^{\sigma}_{t+1}}{\Theta^{\sigma}_t} \;.$$

Taking the log we get $\tau_{t+1} = \sigma\left(\theta_{t+1} - \theta_t\right)$,

resulting in the Backus-Smith condition,

$$C_t = \Theta_t C^i_t \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}.$$

Taking logs on both sides and integrating over *i*, we get

$$c_t = c^*_t + \frac{1}{\sigma} q_t + \theta_t$$

Appendix 3 Uncovered Interest Parity

The pricing equation for foreign bonds and domestic bonds are respectively

$$\left(R_t^*\right)^{-1} = E_t\left\{Q_{t,t+1}^*\right\},$$
$$\left(R_t\right)^{-1} = E_t\left\{Q_{t,t+1}\right\}.$$

We combine them to get the Uncovered Interest Parity conditions,

$$E_t\left\{Q_{t,t+1}R_t - Q_{t,t+1}^*R_t^*\right\} = 0,$$

$$R_t = \left(1+\tau_{t+1}\right)R_t^*\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}.$$

Taking logs on both sides, we get

$$r_t - r_t^* = E_t\left\{\tau_{t+1}\right\} + E_t\left\{e_{t+1}\right\} - e_t,$$

where $e_t \equiv \int_0^1 e_t^i di$ is the log nominal effective exchange rate.

The bilateral terms of trade between the domestic country and country *i* are

$$S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}.$$

The effective terms of trade are

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma}di\right)^{\frac{1}{1-\gamma}}.$$

Taking logs, we get

$s_t \equiv p_{F,t} - p_{H,t}$,

$s_t = \int_0^1 s_{i,t} di$ (when $\gamma = 1$).

Under the purchasing power parity condition, $P_{H,t} = P_{F,t}$, so that $S_t = 1$.

Log linearizing, $P_t \equiv \left[(1-\alpha) \left(P_{H,t} \right)^{1-\eta} + \alpha \left(P_{F,t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}$ becomes

$p_t \equiv (1-\alpha)\, p_{H,t} + \alpha\, p_{F,t} = p_{H,t} + \alpha s_t$, when $\eta = 1$.

It follows that

$\pi_t = \pi_{H,t} + \alpha \left(s_t - s_{t-1} \right)$

and

$E_t \left\{ \pi_{t+1} \right\} = E_t \left\{ \pi_{H,t+1} \right\} + \alpha \left[E_t \left\{ s_{t+1} \right\} - s_t \right]$.

The bilateral nominal exchange rate is defined by the law of one price,

$P_{i,t}(j) = \mathcal{E}_{i,t} P_{i,t}^i(j)$,

where $P_{i,t}^i(j)$ is the price of country *i*’s good *j*, expressed in country *i*’s currency.

It follows that $P_{i,t} = \mathcal{E}_{i,t} P_{i,t}^i$. The nominal effective exchange rate is defined as

$\mathcal{E}_t \equiv \left(\int_0^1 \mathcal{E}_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$.

Log linearizing $P_{F,t} \equiv \left(\int_0^1 P_{i,t}{}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$ and substituting $P_{i,t}$ into $P_{F,t}$, we get

$$p_{F,t} = \int_0^1 \left(e_{i,t} + p_{i,t}^i \right) di = e_t + p_t^* ,$$

where $p_t^* \equiv \int_0^1 p_{i,t}^i di$ is the log world price index. Combining the previous result with terms of trade, we get

$$s_t = e_t + p_t^* - p_{H,t} .$$

The real exchange rate is defined as $\mathcal{Q}_{i,t} \equiv \dfrac{\mathcal{E}_{i,t} P_t^i}{P_t}$.

We can rewrite the uncovered interest parity condition as

$$r_t - r_t^* = E_t \left\{ \tau_{t+1} \right\} + E_t \left\{ e_{t+1} \right\} - e_t .$$

Since $\tau_{t+1} = \sigma \left(\theta_{t+1} - \theta_t \right)$ and $e_t = s_t + p_{H,t} - p_t^*$, it follows that

$$r_t - r_t^* = \sigma \left[E_t \left\{ \theta_{t+1} \right\} - \theta_t \right] + \left[E_t \left\{ s_{t+1} \right\} - s_t \right] + E_t \left\{ \pi_{H,t+1} \right\} - E_t \left\{ \pi_{t+1}^* \right\} .$$

Appendix 4 Equilibrium of Demand Side

The market clearing condition in the representative small open economy is

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j)di$$

$$= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} C_t^i di \right],$$

where the assumption of symmetric preferences across countries produces

$$C_{H,t}^i(j) = \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} C_t^i \ .$$

Substituting into $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$, we get

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta} C_t^i di \right],$$

$$Y_t = S_t^{\alpha} C_t \left[(1-\alpha) + \alpha \Theta_t^{-1} \right] .$$

The first-order log linear approximation is

$$y_t = \alpha s_t + c_t - \theta_t .$$

Substituting this into $c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(r_t - E_t\{\pi_{t+1}\} - \rho\right)$, we get

$$y_t = E_t\{y_{t+1}\} - \left(r_t - E_t\{\pi_{t+1}\} - \rho\right) - \alpha\left[E_t\{s_{t+1}\} - s_t\right] + \left[E_t\{\theta_{t+1}\} - \theta_t\right].$$

Appendix 5 Equilibirum of Supply Side

At the steady state of the economy, we have

$$y_t = a_t + n_t .$$

The real marginal cost is

$$mc_t = -\nu + c_t + \varphi n_t + \alpha s_t - a_t ,$$

while the steady state real marginal cost is

$$mc \equiv -\mu .$$

The deviation of real marginal cost from its steady state is

$$mc_t \equiv mc_t - mc = \mu - \nu + c_t + \varphi n_t + \alpha s_t - a_t = \mu - \nu + (\varphi + 1)(y_t - a_t) + \theta_t .$$

Thus at equilibrium, the dynamic equation for inflation is

$$\pi_{H,t} = \beta E_t \{\pi_{H,t+1}\} + \lambda mc_t = \beta E_t \{\pi_{H,t+1}\} + \lambda(\mu - \nu) + \lambda(\varphi + 1) y_t - \lambda(\varphi + 1) a_t + \lambda \theta_t .$$

Appendix 6 Equilibrium Dynamics in Output Gap

The natural level of output is defined to be the equilibrium output in the absence of nominal rigidities, where the deviation of real marginal cost from its steady state equals 0, as follows:

$$mc_t = 0 \Rightarrow \bar{y}_t = a_t - \frac{1}{\varphi+1}\theta_t + \frac{\nu-\mu}{\varphi+1}.$$

The output gap is defined to be the following deviation of output from its natural level:

$x_t \equiv y_t - \bar{y}_t$, so that

$$y_t = x_t + \bar{y}_t = x_t + \left(a_t - \frac{1}{\varphi+1}\theta_t + \frac{\nu-\mu}{\varphi+1} \right).$$

We substitute that equation into the dynamics of output and inflation and also substitute π_{t+1} into the expression of $\pi_{H,t+1}$ to acquire

$$x_t = E_t\{x_{t+1}\} - \left[r_t - E_t\{\pi_{H,t+1}\} - \rho \right] + \left[E_t\{a_{t+1}\} - a_t \right] + \frac{\varphi}{\varphi+1}\left[E_t\{\theta_{t+1}\} - \theta_t \right],$$

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda(\varphi+1)x_t,$$

together with the uncovered interest parity condition

$$r_t - r_t^* = \left[E_t\{\theta_{t+1}\} - \theta_t \right] + \left[E_t\{s_{t+1}\} - s_t \right] + E_t\{\pi_{H,t+1}\} - E_t\{\pi_{t+1}^*\}.$$

The above three equations constitute the dynamics of the economy with capital controls and flexible exchange rates, but without monetary policy.

If the exchange rate is fixed, then $e_{t+1} = e_t$, so that

$$E_t\left\{\pi_{H,t+1}\right\} = E_t\left\{\pi^*_{t+1}\right\} - \left[E_t\left\{s_{t+1}\right\} - s_t\right],$$

$$r_t - r^*_t = \left[E_t\left\{\theta_{t+1}\right\} - \theta_t\right].$$

When purchasing power parity holds, $S_t = 1$ and $\left[E_t\left\{s_{t+1}\right\} - s_t\right] = 0$.

Appendix 7 Proof of Proposition 1

Under capital control, flexible exchange rates, and current-looking monetary policy, the system can be rewritten as

$$E_t(x_{t+1}) = (1+\frac{a_x}{\varphi+1}+\frac{\lambda}{\beta})x_t - \frac{(1-a_\pi \beta)}{\beta(\varphi+1)}\pi_{H,t} - [E_t(a_{t+1})-a_t] + \frac{\varphi}{\varphi+1}r_t^* - \frac{\varphi}{\varphi+1}E_t(\pi_{t+1}^*) - \rho,$$
$$E_t(\pi_{H,t+1}) = \frac{1}{\beta}\pi_{H,t} - \frac{\lambda(\varphi+1)}{\beta}x_t.$$

The two-dimensional subsystem for the conditional expectations, $\boldsymbol{\xi}_t = \begin{bmatrix} \xi_t^x & \xi_t^{\pi_H} \end{bmatrix}'$, where $\xi_t^x = E_t(x_{t+1})$ and $\xi_t^{\pi_H} = E_t(\pi_{H,t+1})$ can be written as

$$\boldsymbol{\xi}_t = \boldsymbol{\Gamma}_1^* \boldsymbol{\xi}_{t-1} + \boldsymbol{\Psi}^* \boldsymbol{\varepsilon}_t + \boldsymbol{\Pi}^* \boldsymbol{\eta}_t .$$

The eigenvalues for $\boldsymbol{\Gamma}_1^*$ are

$$\mu_1, \mu_2 = \frac{A+\frac{1}{\beta} \pm \sqrt{(A+\frac{1}{\beta})^2 - \frac{4(A-EB)}{\beta}}}{2},$$

where

$$A = 1+\frac{a_x}{\varphi+1}+\frac{\lambda}{\beta},$$
$$B = \frac{(1-a_\pi \beta)}{\beta(\varphi+1)},$$
$$E = \lambda(\varphi+1).$$

Since the number of non-fundamental errors $k = 2$, when $r = m = 1$, there will be one degree of indeterminacy. This requires that only one of the roots, μ_1 and μ_2, be unstable, resulting in this conclusion.

Appendix 8 Proof of Proposition 2

Under capital control, fixed exchange rates, and current-looking monetary policy, the system can be rewritten as

$$E_t(x_{t+1}) = (1 + \frac{a_x}{\varphi+1} + \frac{\varphi+1}{\beta})x_t - (\frac{1}{\beta} - \frac{a_\pi}{\varphi+1})\pi_{H,t} - [E_t(a_{t+1}) - a_t] + \frac{\varphi}{\varphi+1} r_t^* - \rho,$$
$$E_t(\pi_{H,t+1}) = \frac{1}{\beta}\pi_{H,t} - \frac{\lambda(\varphi+1)}{\beta} x_t.$$

The eigenvalues of matrix $\mathbf{\Gamma}_1^*$ are

$$\mu_1, \mu_2 = \frac{A + \frac{1}{\beta} \pm \sqrt{(A + \frac{1}{\beta})^2 - \frac{4(A - EB)}{\beta}}}{2},$$

where

$$A = 1 + \frac{a_x}{\varphi+1} + \frac{\varphi+1}{\beta},$$
$$B = \frac{1}{\beta} - \frac{a_\pi}{\varphi+1},$$
$$E = \lambda(\varphi+1).$$

This result follows.

Appendix 9 Proof of Proposition 3

Under capital control, flexible exchange rates, and forward-looking monetary policy, the system can be rewritten as

$$(1-\frac{a_x}{\varphi+1})E_t(x_{t+1}) = \left[1-\frac{\lambda(a_\pi-1)}{\beta}\right]x_t - \frac{(1-a_\pi)}{\beta(\varphi+1)}\pi_{H,t} - [E_t(a_{t+1})-a_t] + \frac{\varphi}{\varphi+1}r_t^* - \frac{\varphi}{\varphi+1}E_t(\pi_{t+1}^*) - \rho,$$

$$E_t(\pi_{H,t+1}) = \frac{1}{\beta}\pi_{H,t} - \frac{\lambda(\varphi+1)}{\beta}x_t.$$

The eigenvalues of matrix $\mathbf{\Gamma}_1^*$ are

$$\mu_1, \mu_2 = \frac{\frac{A}{F}+\frac{1}{\beta} \pm \sqrt{(\frac{A}{F}+\frac{1}{\beta})^2 - \frac{4(A-EB)}{F\beta}}}{2},$$

where

$$A = 1-\frac{\lambda(a_\pi-1)}{\beta},$$
$$B = \frac{1-a_\pi}{\beta(\varphi+1)},$$
$$E = \lambda(\varphi+1),$$
$$F = 1-\frac{a_x}{\varphi+1}.$$

This result follows.

Appendix 10 Proof of Proposition 4

Under capital control, fixed exchange rates, and forward-looking monetary policy, the system can be rewritten as

$$(1-\frac{a_x}{\varphi+1})E_t(x_{t+1}) = \left[1+\frac{\lambda(\varphi+1-a_\pi)}{\beta}\right]x_t - \left[\frac{1}{\beta}-\frac{a_\pi}{\beta\left(\varphi+1\right)}\right]\pi_{H,t} - [E_t(a_{t+1})-a_t] + \frac{\varphi}{\varphi+1}r_t^* - \rho,$$
$$E_t(\pi_{H,t+1}) = \frac{1}{\beta}\pi_{H,t} - \frac{\lambda(\varphi+1)}{\beta}x_t.$$

The eigenvalues of matrix $\mathbf{\Gamma}_1^*$ are

$$\mu_1, \mu_2 = \frac{\frac{A}{F}+\frac{1}{\beta}\pm\sqrt{(\frac{A}{F}+\frac{1}{\beta})^2-\frac{4(A-EB)}{F\beta}}}{2},$$

where

$$A = 1+\frac{\lambda(\varphi+1-a_\pi)}{\beta},$$
$$B = \frac{1}{\beta}-\frac{a_\pi}{\beta(\varphi+1)},$$
$$E = \lambda(\varphi+1),$$
$$F = 1-\frac{a_x}{\varphi+1}.$$

This result follows.

Appendix 11 Proof of Proposition 5-8

1. Case 1

We rewrite the system in 2×2 form as

$$\begin{bmatrix} E_t(x_{t+1}) \\ E_t(\pi_{H,t+1}) \end{bmatrix} = \begin{bmatrix} A & -B \\ -\dfrac{E}{\beta} & \dfrac{1}{\beta} \end{bmatrix} \begin{bmatrix} x_t \\ \pi_{H,t} \end{bmatrix} + \mathbf{\Psi Z}_t + \mathbf{C},$$

where A, B, E, and $\mathbf{Z}_t$ are defined the same as in Case 1 for indeterminacy. The characteristic equation is

$$\mu^2 - (A + \frac{1}{\beta})\mu + \frac{A - EB}{\beta} = 0.$$

For bifurcation to exist, the following conditions must be satisfied:

$$D = (A + \frac{1}{\beta})^2 - 4\frac{A - EB}{\beta} < 0,$$

$$|\mu_{1,2}| = \sqrt{\theta^2 + \omega^2} = \sqrt{\mu_1 \mu_2} = \sqrt{\frac{A - EB}{\beta}} = 1.$$

This result follows.

2. Case 2

We again rewrite the system in 2×2 form as equations (34), but with A, B, E, and $\mathbf{Z}_t$ defined as in Case 2 for indeterminacy. The characteristic equation and the bifurcation condition equations are the same as in Case 1, but with the different settings of A, B, E, and $\mathbf{Z}_t$.

This result follows.

3. Case 3

We again rewrite the system in 2×2 form as equations (34), but with A, B, E, and $\mathbf{Z}_t$ defined as in Case 3 for indeterminacy. The characteristic equation is

$$\mu^2 - (\frac{A}{F} + \frac{1}{\beta})\mu + \frac{A - EB}{F\beta} = 0.$$

For bifurcation to exist, the following conditions must be satisfied.

$$D = (\frac{A}{F} + \frac{1}{\beta})^2 - 4\frac{A - EB}{F\beta} < 0,$$

$$\left|\mu_{1,2}\right| = \sqrt{\theta^2 + \omega^2} = \sqrt{\mu_1\mu_2} = \sqrt{\frac{A - EB}{F\beta}} = 1.$$

This result follows.

4. Case 4

We again rewrite the system in 2×2 form as equations (34), but with A, B, E, and $\mathbf{Z}_t$ defined the same as in Case 4 for indeterminacy. The characteristic equation and the bifurcation condition equations are the same as in Case 3, but with different settings of A, B, E, and $\mathbf{Z}_t$.

This result follows.

Chapter 2

Exchange Rate as a Diffusion Process

2.1. Introduction

Under the floating exchange rate regime, exchange rate is known as highly volatile, nonstationary. It is also hard to be explained and predicted by fundamentals, such as output, price level and monetary supply. This paper is trying to use a continuous time diffusion process to model the above characters of exchange rate dynamics. Both nonparametric method and parametric method (MLE) are used to estimate exchange rate as a diffusion process. In line with previous literature's finding, the nonparametric drift estimators show some nonlinearity. The result of parametric estimation shows that Geometric Brownian Motion process could be a quite good capture of the exchange rate dynamics.

2.2. Litereture Review

Firstly, for the estimation of diffusion process, lots of existing literature have estimated the interest rates or the prices of derivative securities as diffusion processes. Earlier diffusion models have to rely on parametric or semi-parametric specifications for the drift and diffusion functions in order to implement available estimation methods based on discretely observed data. However, both parametric and semiparametric specifications impose very strong and unrealistic assumptions on the underlying process of the model. With the development of estimation methods, Ait-Sahalia (1996) proposed a nonparametric diffusion function estimator based on the linear mean-reverting drift function for the strictly stationary diffusion processes. Jiang and Knight (1997), Stanton (1997) assumed both drift function and diffusion function are unknown and used nonparametric method to estimate them. In addition, Stanton (1997) found evidence of substantial nonlinearity in the drift. Jiang and Knight (1997, 2nd) performed

Monte Carlo simulation to compare parametric versus nonparametric estimation of diffusion process.

Secondly, for the estimation of exchange rate, Cai et al (2012) used functional-coefficient model for the conditional mean and a GARCH type model with a policy dummy variable for the conditional volatility. They found that the USD/CNY exchange rate is nonstationary and has an obvious decreasing time trend. So they chose the return of exchange rate (the scaled logarithm difference) as the objective to study.

2.3. Model and Data

Observed from the exchange rate data under a floating regime, the exchange rate is highly volatile and nonstationary, but without an obvious increasing or decreasing time trend. Thus a diffusion process might be a good capture of these characters. My main objective here is to study the exchange rate dynamics (E_t). However, I will still show a description of the return series (ln (E_t)-ln (E_{t-1})). In addition, when the Maximum Likelihood method is used in the estimation of drift and diffusion parameters for the exchange rate process, we also need the help of the return series.

A general diffusion process considered is

$$dE_t = \mu(E_t)dt + \sigma(E_t)dW_t$$

Kernel density estimation is used to estimate the density of exchange rate. The drift function and diffusion function are estimated by Nadaraya-Watson nonparametric estimation. The bandwidth is calculated using Scott's rule (i.e., bw = len^(-1/5)*sd(x)), where len=length(x) is the number of observed points of the diffusion path. Besides Scott's rule, I also use other bandwidth values to compare the differences.

Since exchange rate series is nonstationary, there exist problems on the nonparametric estimation of drift function as stated in Jiang and Knight (1997, 2nd). *"The nonparametric drift function estimator requires stronger conditions and the stochastic process must be at least asymptotically stationary in the strict sense. This excludes the application of the proposed nonparametric drift function estimator to such processes as Brownian motion with drift and geometric Brownian motion. Both the Brownian motion with drift process and the geometric Brownian motion process are nonstationary. Since the Brownian motion with drift process is neither strictly stationary nor has a limiting probability density function, the aforementioned nonparametric drift function estimator cannot be applied."*

Thus if we assume the nonstationary exchange rate series follows the Geometric Brownian Motion process (drift function and diffusion function are both linear),

$$dE_t = \mu E_t dt + \sigma E_t dW_t$$

Parametric method should be used to estimate the drift and diffusion. I will use the Maximum likelihood method to estimate μ and σ.

The Maximum likelihood estimation procedure is as follows,

Solution to the Stochastic Differential Equation of GBM is $E_t = E_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t}$

Which can be written as $\ln(E_t) = \ln(E_0) + (\mu - \frac{1}{2}\sigma^2)t + \sigma W_t$

Thus $\ln(E_{t_k}) - \ln(E_{t_{k-1}}) = (\mu - \frac{1}{2}\sigma^2)(t_k - t_{k-1}) + \sigma(W_{t_k} - W_{t_{k-1}})$

And $\ln(E_{t_k}) - \ln(E_{t_{k-1}}) \sim N((\mu - \frac{1}{2}\sigma^2)(t_k - t_{k-1}), \sigma^2(t_k - t_{k-1}))$

(Test will be performed to see whether return series is independent and normal.)

Let $x_{t_k} \equiv \ln(E_{t_k}) - \ln(E_{t_{k-1}})$ and $\Delta_{t_k} \equiv t_k - t_{k-1}$

The maximum likelihood function is $L(\theta) = \sum_{k=1}^{n} \ln(f_\theta(x_{t_k}))$

Where $f_\theta(x_{t_k}) = \frac{1}{E_{t_k}\sigma\sqrt{2\pi\Delta_{t_k}}} \exp(-\frac{[x_{t_k} - (\mu - \frac{1}{2}\sigma^2)\Delta_{t_k}]^2}{2\sigma^2\Delta_{t_k}})$

We have the mean $\hat{m} \equiv (\hat{\mu} - \frac{1}{2}\hat{\sigma}^2)\Delta_{t_k}$ and the variance $\hat{v} \equiv \hat{\sigma}^2\Delta_{t_k}$

Since $\hat{m} = \sum_{k=1}^{n} \frac{x_{t_k}}{n}$ and $\hat{v} = \sum_{k=1}^{n} \frac{(x_{t_k} - \hat{m})^2}{n}$

So $\hat{\mu} = \hat{m}n + \frac{1}{2}\hat{\sigma}^2$ and $\hat{\sigma} = \sqrt{\hat{v}n}$

After the MLE estimation of drift and diffusion, simulation is performed to see whether the simulated exchange rate series under Geometric Brownian Motion process is close to the real data.

The data used in this paper is the nominal exchange rate indices (calculated as geometric weighted averages of bilateral exchange rates) by Bank of International Settlements. I used the United States' daily data from October 3rd, 1983 to November 28th, 2016.

The dynamics of exchange rate is shown in Figure 18 and its summary description is provided in Table 38. The return series is shown in Figure 19 and its description is summarized in Table 39. Phillips–Perron test is performed and we can conclude that the exchange rate series is nonstationary and the return series is stationary.

Figure 18: Exchange Rate

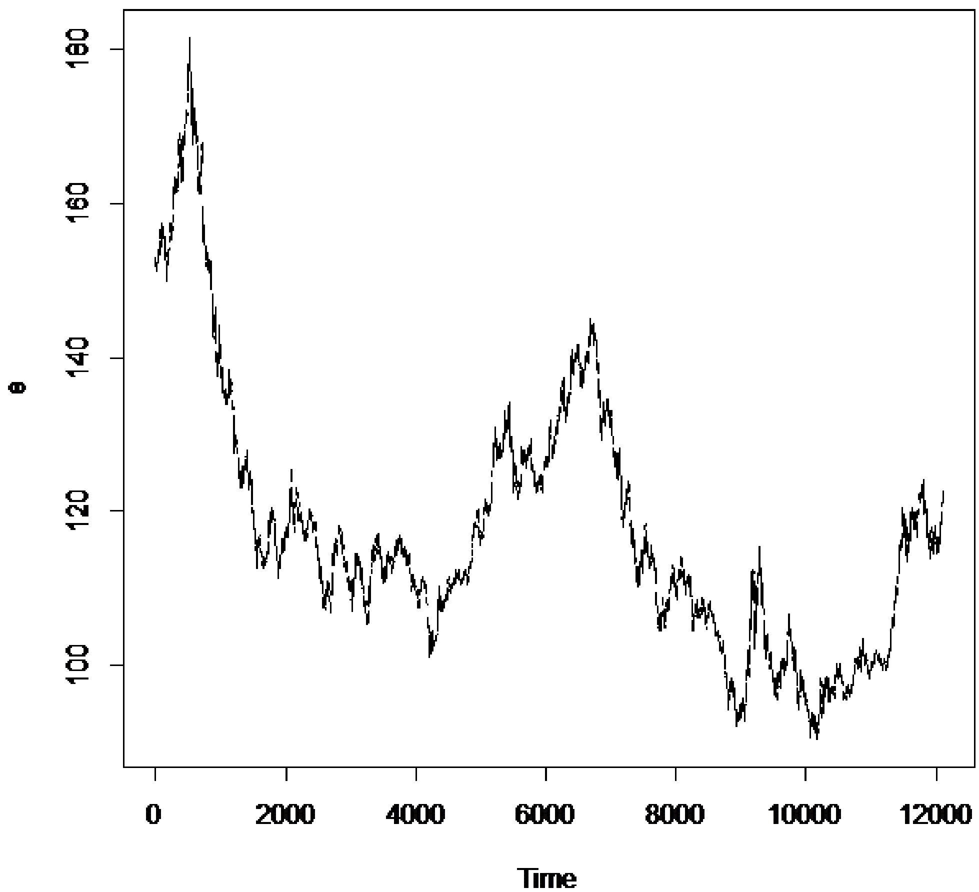


Table 38: Summary of Exchange Rate

	Summary statistics
Min	90.5
Max	181.5
Mean	117.6
Variance	288.9973
Skewness	1.169896
Kurtosis	1.405656

Figure 19: Return ln (Et)-ln (Et-1)

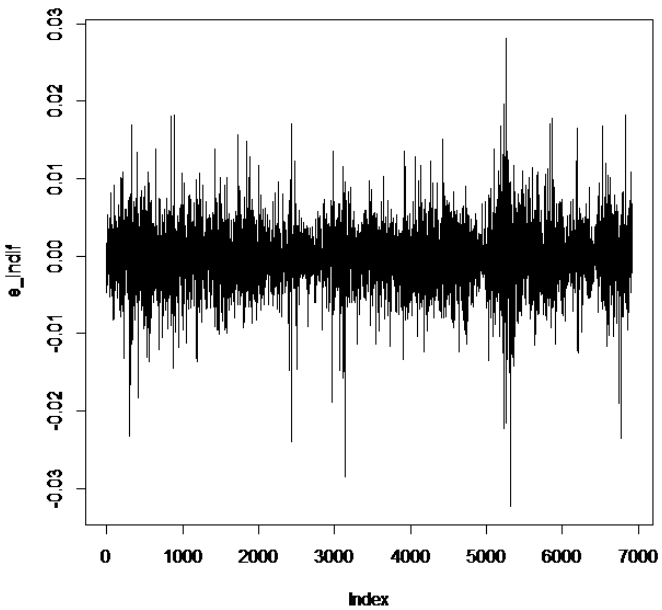


Table 39: Summary of Return ln (Et)-ln (Et-1)

Summary statistics	
Min	-3.222e-02
Max	2.807e-02
Mean	-5.116e-05
Variance	1.502843e-05
Skewness	-0.2021381
Kurtosis	6.869734

The Phillips-Perron unit root test for exchange rate shows the following results.

Title: Phillips-Perron Unit Root Test

Test regression with intercept

Residuals:

Min	1Q	Median	3Q	Max
-6.0655	-0.2521	0.0035	0.2668	2.9064

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.0721580	0.0361971	1.993	0.0462 *
y.l1	0.9993564	0.0003047	3279.849	<2e-16 ***

Signif.codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1‘ ’1

Residual standard error:0.4817 on 8648 degrees of freedom

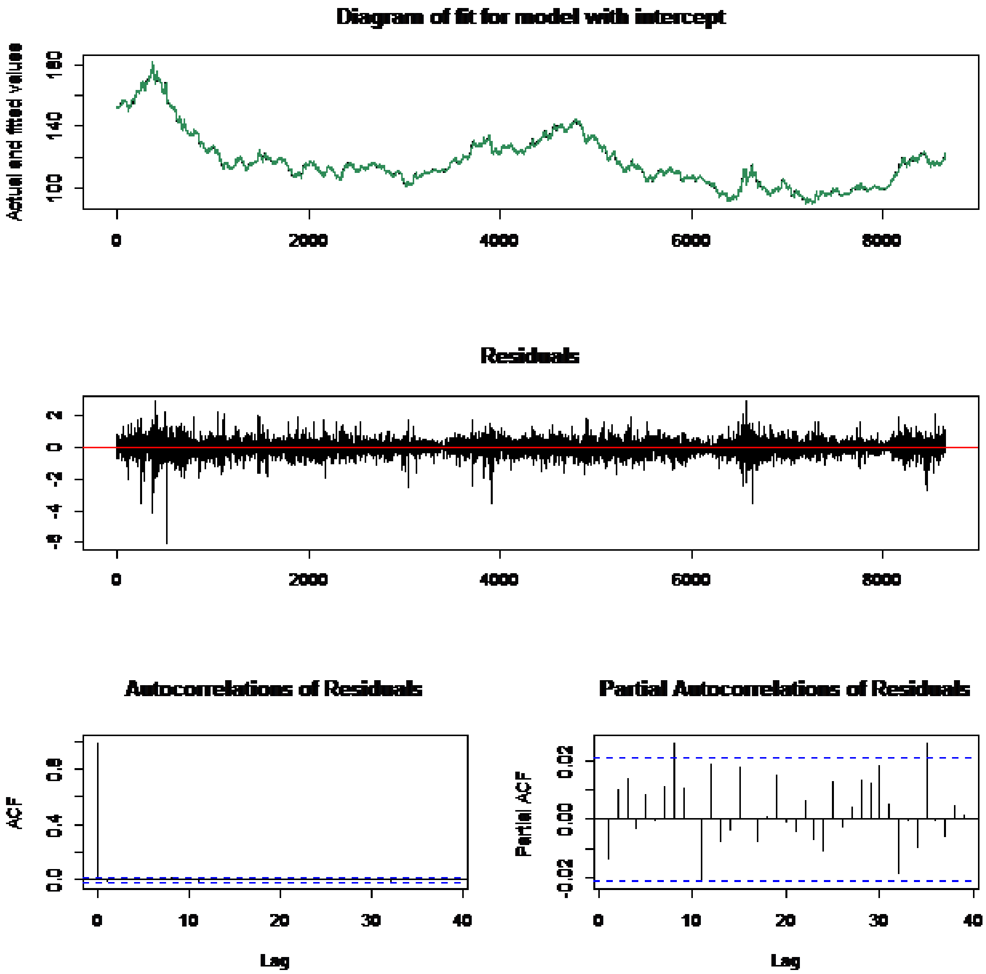
Multiple R-squared:0.9992, Adjusted R-squared:0.9992

F-statistic: 1.076e+07 on 1 and 8648 DF, p-value: < 2.2e-16

Value of test-statistic, type: Z-alpha is: -5.7576

aux. Z statistics

Z-tau-mu 2.0106



The Phillips-Perron unit root test for return series shows the following results.

Title: Phillips-Perron Unit Root Test

Test regression with intercept

Residuals:

Min	1Q	Median	3Q	Max
-0.032183	-0.002116	0.000048	0.002169	0.028088

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-5.150e-05	4.662e-05	-1.105	0.269
y.l1	-1.707e-03	1.202e-02	-0.142	0.887

Residual standard error: 0.003877 on 6917 degrees of freedom

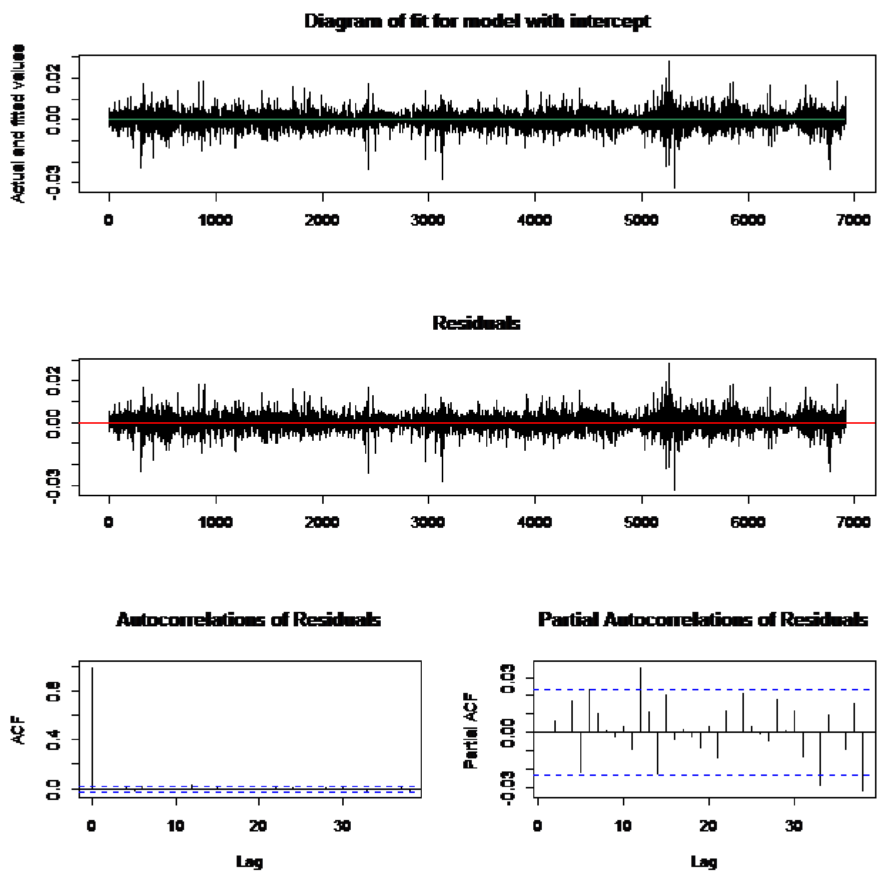
Multiple R-squared: 2.913e-06, Adjusted R-squared: -0.0001417

F-statistic:0.02015 on 1 and 6917 DF, p-value: 0.8871

Value of test-statistic, type: Z-alpha is: -7073.474

aux. Z statistics

Z-tau-mu -1.1048



2.4. Estimation

2.4.1. Nonparametric estimation of exchange rate

Assume exchange rate follows the diffusion process

$$dE_t = \mu(E_t)dt + \sigma(E_t)dW_t\,.$$

Figure 20: Kernel Density (Bandwidth = 2.774)

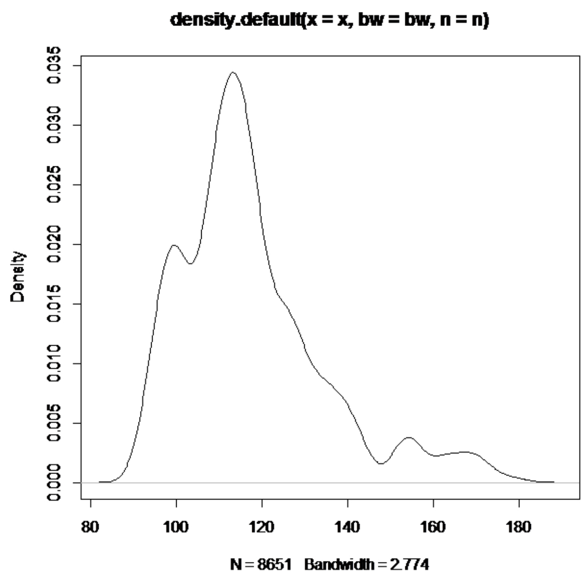


Figure 21: Drift Function (Bandwidth = 2.774)

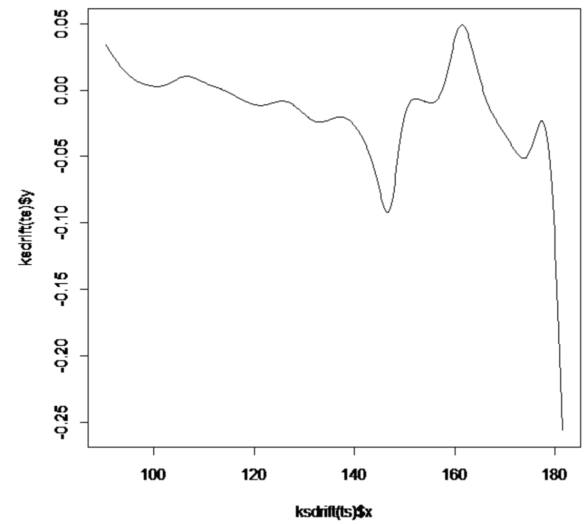


Figure 22: Diffusion Function (Bandwidth = 2.774)

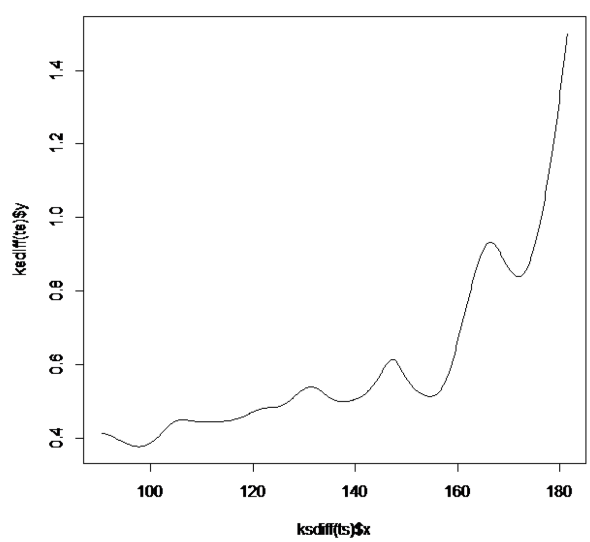


Figure 23: Kernel Density (Bandwidth = 5)

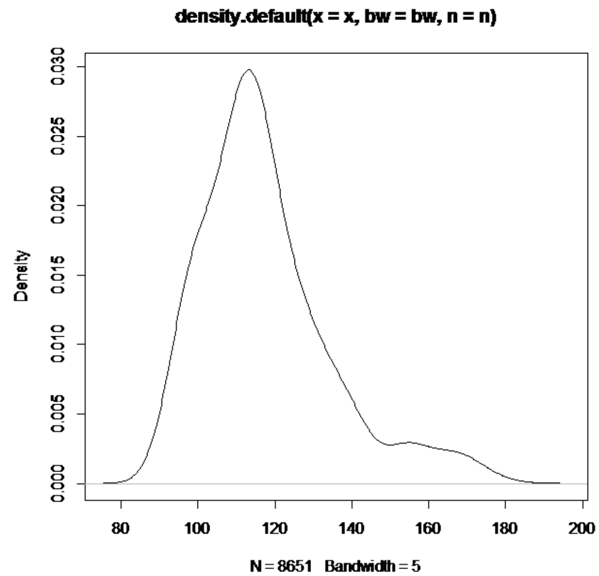


Figure 24: Drift Function (Bandwidth = 5)

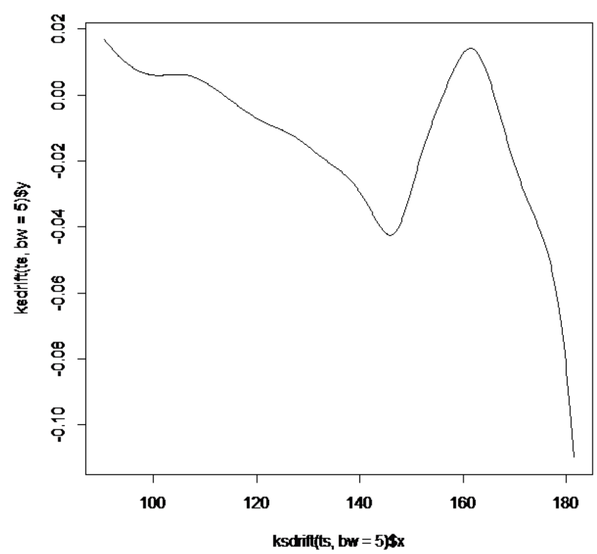
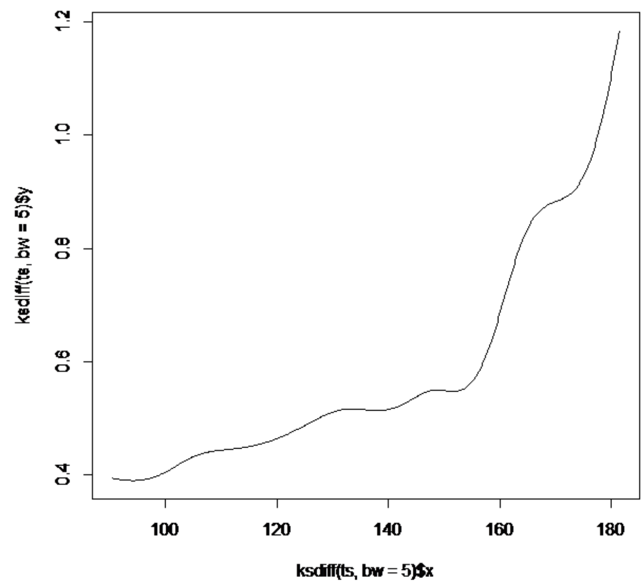


Figure 25: Diffusion Function (Bandwidth = 5)



2.4.2. Estimation as Geometric Brownian Motion by MLE

Assume exchange rate follows the Geometric Brownian Mothion

$dE_t = \mu E_t dt + \sigma E_t dW_t$.

First, we need to test whether series of ln (E_t)-ln (E_{t-1}) is independent and normal.

Figure 26: ACF of ln (Et)-ln (Et-1)

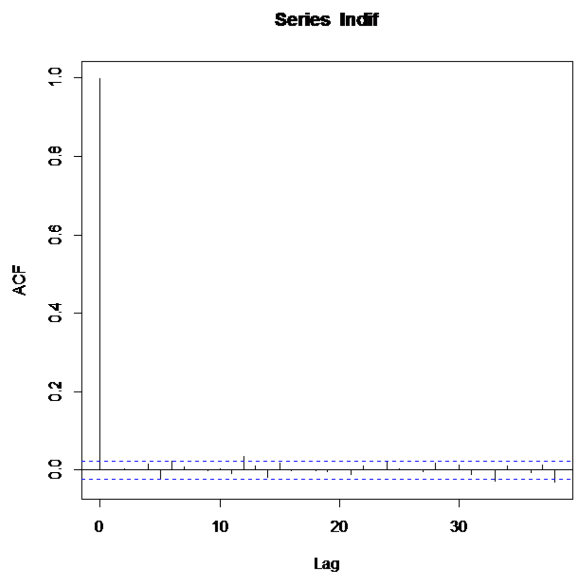


Figure 27: PACF of ln (Et)-ln (Et-1)

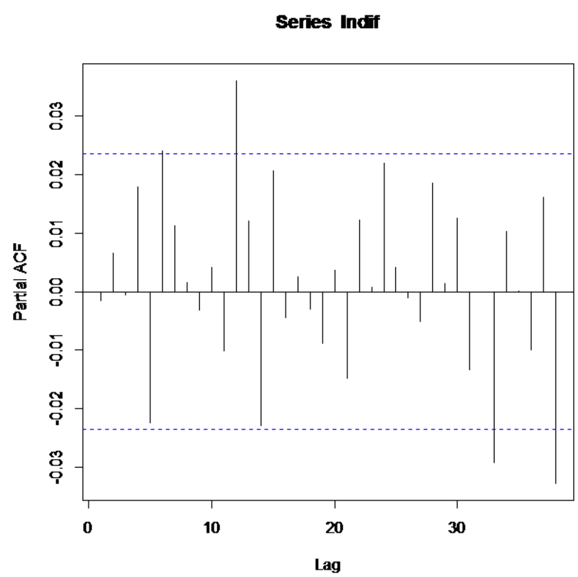
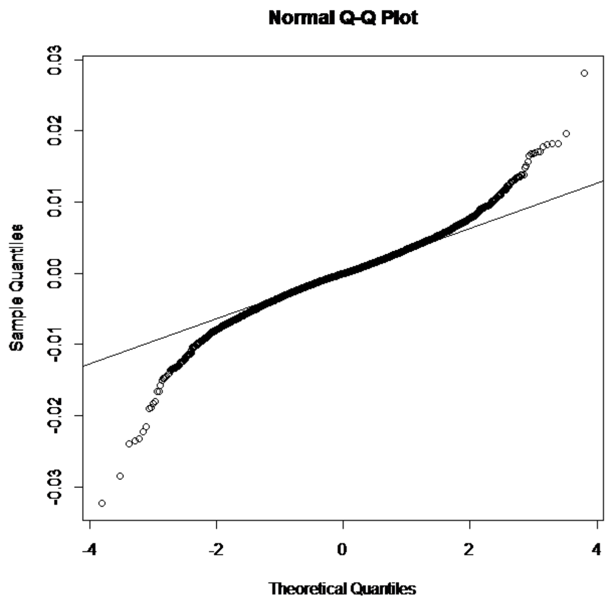


Figure 28: QQ-plot of ln (Et)-ln (Et-1)



Then, according to the MLE estimation procedure in section 2.3,

$$\hat{\mu} = \hat{m}n + \frac{1}{2}\hat{\sigma}^2 = -0.5285348$$

$$\hat{\sigma} = \sqrt{\hat{v}n} = 0.4266255$$

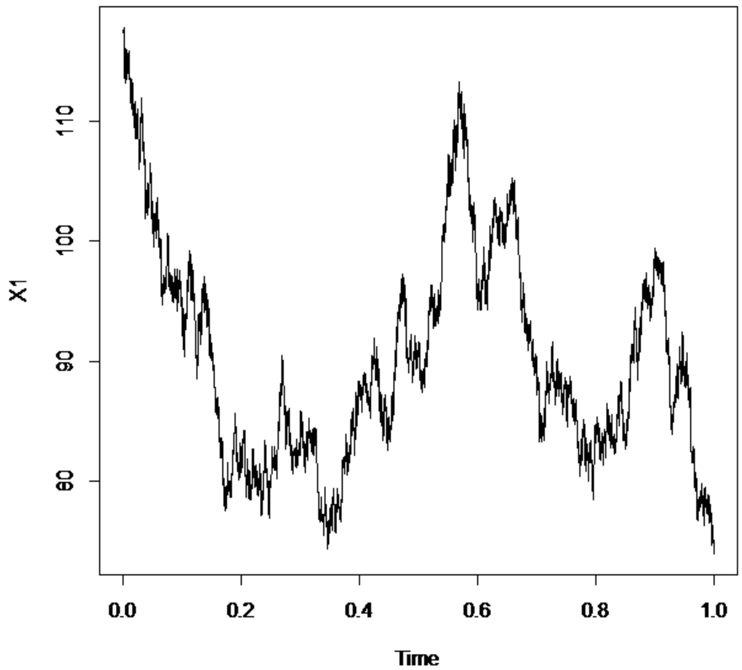
Thus

$$dE_t = (-0.5285348)E_t dt + (0.4266255)E_t dW_t$$

2.4.3. Simulation as Geometric Brownian Motion process

Let E_0 = 117.6, μ= -0.5, σ= 0.4.

Figure 29: Simulation result of GBM process



We can see that the trend is similar to that of the real data. Thus Geometric Brownian Motion process is a good capture of the exchange rate dynamics.

2.4.4. Nonparametric estimation of return

Assume the reurn $\ln(E_t)$-$\ln(E_{t-1})$ follows the diffusion process

$$dE_t = \mu(E_t)dt + \sigma(E_t)dW_t.$$

When bandwidth=0.5

Figure 30: Kernel Density (Bandwidth = 0.5)

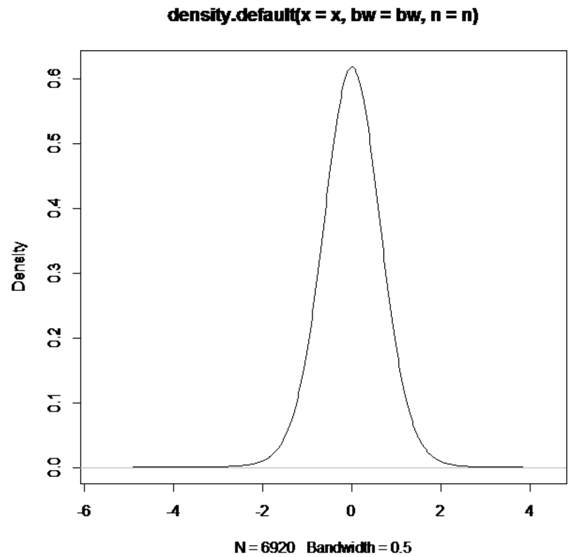


Figure 31: Drift Function (Bandwidth = 0.5)

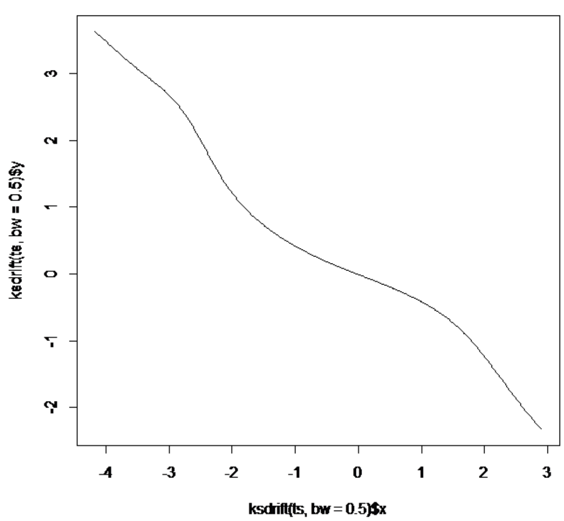
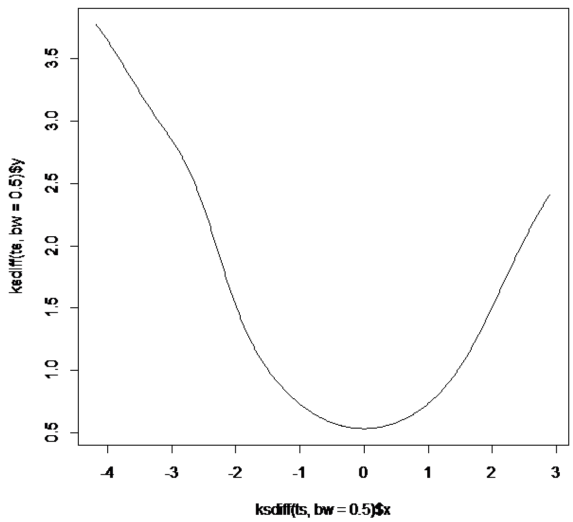


Figure 32: Diffusion Function (Bandwidth = 0.5)



2.5. Conclusion

To sum up, in this paper, I use both nonparametric method and parametric method (MLE) to estimate exchange rate as a diffusion process. In line with the previous literature's finding, the nonparametric drift estimators show some nonlinearity. This may be because that the exchange rate series is nonstationary. The result of parametric estimation shows that Geometric Brownian Motion process could be a quite good capture of the exchange rate dynamics.

2.6. References

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